



On Fuzzy Semi-P-spaces and Related Concepts

Ganesan Thangaraj¹, Ayyavu Vinothkumar²

¹Department of Mathematics, Thiruvalluvar University, Vellore, India

²Research Scholar, Department of Mathematics, Thiruvalluvar University, Vellore, India

Email address:

g.thangaraj@rediffmail.com (G. Thangaraj), thamizhvino1@gmail.com (A. Vinothkumar)

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Abstract: In this paper the notion of fuzzy semi-P-spaces is introduced and studied. It is established that the class of fuzzy semi-P-spaces lies between the classes of fuzzy P-spaces and fuzzy almost-P-spaces. It is established that fuzzy σ -nowhere dense sets in fuzzy Semi-P-spaces are fuzzy Semi-closed sets and fuzzy G_δ -sets and fuzzy F_σ -sets are fuzzy σ -nowhere sets in fuzzy hyperconnected and semi-P-spaces are fuzzy dense sets. Also it is found that fuzzy residual sets in fuzzy globally disconnected and fuzzy Semi-P-spaces are fuzzy semi-open sets and fuzzy G_δ -sets in fuzzy perfectly disconnected and fuzzy Semi-P-spaces are fuzzy pre-open sets. Also it is established that fuzzy hyperconnected and semi-P-spaces are fuzzy irresolvable spaces. The conditions for fuzzy semi-P-spaces to become fuzzy σ -Baire spaces and fuzzy Baire spaces are obtained. The conditions for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces are also obtained in this paper.

Keywords: Fuzzy G_δ -set, Fuzzy F_σ -set, Fuzzy Semi-open Set, Fuzzy Almost P-space, Fuzzy σ -Baire Space, Fuzzy Baire Space, Fuzzy Perfectly Disconnected Space

1. Introduction

The concept of fuzzy sets as a new approach for modeling uncertainties was introduced by L. A. Zadeh [21] in the year 1965. The concept of fuzzy topological spaces was introduced by C. L. Chang [4] in 1968. Chang's works paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of fuzzy P-spaces in fuzzy setting was introduced by G. Balasubramanian [7]. Fuzzy Almost P-spaces and Fuzzy GID – spaces were introduced and studied by G. Thangaraj and C. Anbazhagan in [13, 14].

In this paper, the notion of fuzzy semi-P-space is introduced by means of fuzzy semi-openness of fuzzy G_δ -sets. Several characterizations of fuzzy semi-P-spaces are established. It is established that the class of fuzzy semi-P-spaces lies between the classes of fuzzy P-spaces and fuzzy almost P-spaces. Also it is obtained that fuzzy F_σ –sets in fuzzy semi-P-spaces are fuzzy σ -dense sets but not fuzzy dense sets and fuzzy F_σ -sets in fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy σ -nowhere dense sets. Also it is obtained that fuzzy residual sets in fuzzy globally

disconnected and fuzzy semi-P-spaces, are fuzzy semi-open sets. The conditions for the fuzzy hyperconnected and fuzzy semi-P-spaces to become fuzzy σ -Baire spaces and for the fuzzy hyper connected, fuzzy open hereditarily irresolvable and fuzzy semi-P-spaces to become fuzzy Baire spaces are obtained. The conditions for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces and for the fuzzy semi-P-spaces to become fuzzy P-spaces are also obtained in this paper.

2. Preliminaries

Some basic notions and results used in the sequel are given in order to make the exposition self - contained. In this work by (X, T) or simply by X , we will denote a fuzzy topological space due to Chang (1968). Let X be a non - empty set and I the unit interval $[0, 1]$. A fuzzy set λ in X is a mapping from X into I . The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [4]: Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T) . The interior, the closure and

the complement of λ are defined respectively as follows:

- (i). $\text{int}(\lambda) = \bigvee \{ \mu / \mu \leq \lambda, \mu \in T \};$
- (ii). $\text{cl}(\lambda) = \bigwedge \{ \mu / \lambda \leq \mu, 1 - \mu \in T \}.$
- (iii). $\lambda'(x) = 1 - \lambda(x),$ for all $x \in X.$

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T) , the union $\psi = \bigvee_i (\lambda_i)$ and intersection $\delta = \bigwedge_i (\lambda_i)$, are defined respectively as

- (iv). $\psi(x) = \sup_i \{ \lambda_i(x) / x \in X \}$
- (v). $\delta(x) = \inf_i \{ \lambda_i(x) / x \in X \}.$

Lemma 2.1 [1]: For a fuzzy set λ of a fuzzy topological space X ,

- (i). $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$ and (ii). $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda).$

Definition 2.2: A fuzzy set λ in a fuzzy topological space (X, T) is called an

- (i). fuzzy regular-open if $\lambda = \text{int cl}(\lambda)$ and fuzzy regular-closed if $\lambda = \text{cl int}(\lambda)$ [1].
- (ii). fuzzy pre-open if $\lambda \leq \text{int cl}(\lambda)$ and fuzzy pre-closed if $\text{cl int}(\lambda) \leq \lambda$ [3].
- (iii). fuzzy semi-open if $\lambda \leq \text{cl int}(\lambda)$ and fuzzy semi-closed if $\text{int cl}(\lambda) \leq \lambda$ [1].
- (iv). fuzzy β -open if $\lambda \leq \text{cl int cl}(\lambda)$ and fuzzy β -closed if $\text{int cl int}(\lambda) \leq \lambda$ [3].
- (v). fuzzy G_δ -set if $\lambda = \bigwedge_{i=1}^\infty (\lambda_i)$, where $\lambda_i \in T$ for $i \in I$ [2].
- (vi). fuzzy F_σ -set if $\lambda = \bigvee_{i=1}^\infty (\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [2].

Definition 2.3: The fuzzy set λ in an fuzzy topological space (X, T) , is called an

- (i). fuzzy dense set if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is $\text{cl}(\lambda) = 1$, in (X, T) [8].
- (ii). fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) = 0$, in (X, T) [8].
- (iii). fuzzy first category set if $\lambda = \bigvee_{i=1}^\infty (\lambda_i)$, where (λ_i) 's are fuzzy nowhere dense sets in (X, T) . Any other fuzzy set in (X, T) is said to be of fuzzy second category [8].
- (iv). fuzzy somewhere dense set if there exists a non-zero fuzzy open set μ in (X, T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) \neq 0$, in (X, T) [9].
- (v). fuzzy cs dense set if $1 - \lambda$ is an fuzzy somewhere dense set in (X, T) [16].
- (vi). fuzzy σ - nowhere dense set if λ is a fuzzy F_σ -set with $\text{int}(\lambda) = 0$, in (X, T) [12].
- (vii). fuzzy residual set if $1 - \lambda$ is an fuzzy first category set in (X, T) [11].
- (viii). fuzzy σ -boundary set if $\lambda = \bigvee_{i=1}^\infty (\mu_i)$, where $\mu_i = \text{cl}(\lambda_i) \wedge (1 - \lambda_i)$ and (λ_i) 's are fuzzy regular open sets in (X, T) [15].
- (ix). fuzzy co - σ boundary set if $1 - \lambda$ is an fuzzy σ -boundary set in (X, T) [15].
- (x). Fuzzy simply open set if $\text{Bd}(\lambda)$ is a fuzzy nowhere dense set in (X, T) . That is, λ is a fuzzy simply open set in (X, T) if $[\text{cl}(\lambda) \wedge \text{cl}(1 - \lambda)]$ is a fuzzy nowhere dense set in (X, T) [18].

Definition 2.4: A fuzzy topological space (X, T) is called an

- (i). fuzzy P-space if each fuzzy G_δ -set in (X, T) is fuzzy open in (X, T) [7].
- (ii). fuzzy almost P - space if for every non- zero fuzzy G_δ -set λ in (X, T) , $\text{int}(\lambda) \neq 0$ in (X, T) [13].
- (iii). fuzzy globally disconnected space if each fuzzy semi-open set in (X, T) is fuzzy open in (X, T) [17].
- (iv). fuzzy hyperconnected space if every non- null fuzzy open subset of (X, T) is fuzzy dense in (X, T) [6].
- (v). fuzzy perfectly disconnected space if for any two non - zero fuzzy sets λ and μ defined on X with $\lambda \leq 1 - \mu$, $\text{cl}(\lambda) \leq 1 - \text{cl}(\mu)$, in (X, T) [18].
- (vi). fuzzy strongly irresolvable space if $\text{cl int}(\lambda) = 1$, for each fuzzy dense set λ in (X, T) [19].
- (vii). fuzzy submaximal space if for each fuzzy set λ in (X, T) such that $\text{cl}(\lambda) = 1$, then $\lambda \in T$ [2].
- (viii). fuzzy Baire space if $\text{int}(\bigvee_{i=1}^\infty (\lambda_i)) = 0$, where (λ_i) 's are fuzzy nowhere dense sets in (X, T) [11].
- (ix). fuzzy σ -Baire space $\text{int}(\bigvee_{i=1}^\infty (\lambda_i)) = 0$, where (λ_i) 's are fuzzy σ - nowhere dense sets in (X, T) [12].
- (x). fuzzy open hereditarily irresolvable space if $\text{int cl}(\lambda) \neq 0$, for any non-zero fuzzy set λ defined on X , then $\text{int}(\lambda) \neq 0$, in (X, T) [10].
- (xi). fuzzy resolvable space if there exists an fuzzy dense set λ in (X, T) such that $\text{cl}(1 - \lambda) = 1$. Otherwise (X, T) is called an fuzzy irresolvable space [10].
- (xii). Fuzzy GID space if for each fuzzy dense and fuzzy G_δ -set λ in (X, T) , $\text{cl int}(\lambda) = 1$, in (X, T) [14].

Definition 2.5 [20]: Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T) . The fuzzy semi-closure and the fuzzy semi-interior of λ are defined as follows:

- (1) $\text{scl}(\lambda) = \bigwedge \{ \mu / \lambda \leq \mu, \mu \text{ is fuzzy semi closed in } (X, T);$
- (2) $\text{sint}(\lambda) = \bigvee \{ \mu / \mu \leq \lambda, \mu \text{ is fuzzy semi open in } (X, T). \}$

Theorem 2.1 [17]: If (X, T) is the fuzzy globally disconnected space, then for any fuzzy semi-closed set λ in (X, T) , $\text{cl}(1 - \lambda) = 1 - \text{cl int}(\lambda)$, in (X, T) .

Theorem 2.2 [13]: If γ is an fuzzy residual set in an fuzzy almost P-space (X, T) , then there exists an fuzzy G_δ -set μ in (X, T) such that $\mu \leq \gamma$.

Theorem 2.3 [16]: If λ is an fuzzy somewhere dense set in an fuzzy topological space (X, T) , then there exist a fuzzy regular closed set η in (X, T) such that $\eta \leq \text{cl}(\lambda)$.

Theorem 2.4 [16]: If λ is an fuzzy somewhere dense set in a fuzzy hyperconnected space (X, T) , then λ is an fuzzy dense set in (X, T) .

Theorem 2.5 [17]: If λ is an fuzzy residual set in the globally disconnected space (X, T) , then λ is an fuzzy G_δ -set in (X, T) .

Theorem 2.6 [11]: If (X, T) is an fuzzy Baire space, then (X, T) is an fuzzy second category space.

Theorem 2.7 [20]: If λ is an fuzzy σ -boundary set in the fuzzy topological space (X, T) , then λ is an fuzzy F_σ -set in (X, T) .

Theorem 2.8 [18]: If λ is an fuzzy σ -nowhere dense set in the fuzzy perfectly disconnected space (X, T) , then there

exists an fuzzy G_δ - set η in (X, T) such that $\lambda \leq \text{int}(1 - \eta)$.

Theorem 2.9 [18]: If λ is an fuzzy co - σ -boundary set in the fuzzy perfectly disconnected space (X, T) , then there exists an fuzzy G_δ -set δ in (X, T) such that $\lambda \leq \delta$.

Theorem 2.10 [18]: If λ is a fuzzy semi-closed set in a fuzzy perfectly disconnected space (X, T) , then λ is a fuzzy pre-closed set in (X, T) .

Theorem 2.11 [14]: Let (X, T) be a fuzzy topological space. Then the following are equivalent:

- (i). (X, T) is a fuzzy GID space.
- (ii). Each fuzzy dense and fuzzy G_δ -set in (X, T) , is fuzzy semi-open in (X, T) .

Theorem 2.12 [17]: If λ is a fuzzy semi-open and fuzzy dense set in the fuzzy globally disconnected space (X, T) , then λ is the fuzzy simply open set in (X, T) .

3. Fuzzy Semi-P-spaces

Definition 3.1: A fuzzy topological space (X, T) is called an fuzzy semi- P-space if each fuzzy G_δ -set in (X, T) is an fuzzy semi-open set in (X, T) . That is (X, T) is an fuzzy semi- P-space if whenever λ is an fuzzy G_δ -set in (X, T) , then $\lambda \leq \text{cl int}(\lambda)$, in (X, T) .

Example 3.1: Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets α, β and γ are defined on X as follows:

$\alpha: X \rightarrow I$ is defined by $\alpha(a)=0.6; \alpha(b)=0.3; \alpha(c)=0.4$,

$\beta: X \rightarrow I$ is defined by $\beta(a)=0.4; \beta(b)=0.7; \beta(c)=0.6$,

$\gamma: X \rightarrow I$ is defined by $\gamma(a)=0.5; \gamma(b)=0.5; \gamma(c)=0.5$,

Then

$T =$

$\{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, 1\}$ is the fuzzy topology on X . On computation, $\gamma = (\alpha \vee \beta) \wedge (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$, $\alpha \wedge \beta = \alpha \wedge \beta \wedge \gamma \wedge (\alpha \wedge \beta) \wedge (\alpha \wedge \gamma) \wedge (\beta \wedge \gamma)$. And thus γ and $\alpha \wedge \beta$ are fuzzy G_δ -sets in (X, T) . Also $\text{cl int}(\gamma) = \text{cl}(\gamma) = 1 - \gamma$ and $\text{cl int}(\alpha \wedge \beta) = \text{cl}(\alpha \wedge \beta) = 1 - (\alpha \vee \beta)$. Now $\gamma \leq 1 - \gamma$ and $\alpha \wedge \beta \leq 1 - (\alpha \vee \beta)$, implies that $\gamma \leq \text{cl int}(\gamma)$ and $(\alpha \wedge \beta) \leq \text{cl int}(\alpha \wedge \beta)$, in (X, T) . Hence the fuzzy G_δ -sets γ and $\alpha \wedge \beta$ are fuzzy semi-open sets in (X, T) , implies that (X, T) is an fuzzy semi- P-space.

Proposition 3.1: If λ is an fuzzy G_δ -set in the fuzzy semi-P-space (X, T) , then

- (i). $\text{int}(\lambda) \neq 0$, in (X, T) .
- (ii). λ is an fuzzy somewhere dense set in (X, T) .
- (iii). $\text{cl int}(\lambda) = \text{cl}(\lambda)$, in (X, T) .
- (iv). λ is an fuzzy β -open set in (X, T) .

Proof:

- (i). Let λ be an fuzzy G_δ -set in (X, T) . Since (X, T) is an fuzzy semi-P-space, the fuzzy G_δ -set λ is an fuzzy semi-open set (X, T) . Then, $\lambda \leq \text{cl int}(\lambda)$, in (X, T) . This implies that $\text{int}(\lambda) \neq 0$, in (X, T) . [otherwise, if $\text{int}(\lambda) = 0$, $\text{cl int}(\lambda) = \text{cl}(0) = 0$ and this results in $\lambda = 0$, a contradiction].
- (ii). By (i), $\text{int}(\lambda) \neq 0$, in (X, T) . Since $\text{int}(\lambda) \leq \text{int cl}(\lambda)$, $\text{int cl}(\lambda) \neq 0$, in (X, T) . Hence λ is an fuzzy somewhere dense set in (X, T) .
- (iii). Since (X, T) is an fuzzy semi-P-space, the fuzzy G_δ -set λ is an fuzzy semi-open set in (X, T) . Then, $\lambda \leq \text{cl}$

$\text{int}(\lambda)$, in (X, T) . This implies that $\text{cl}(\lambda) \leq \text{cl}[\text{cl int}(\lambda)]$ and $\text{cl}(\lambda) \leq \text{cl int}(\lambda)$. But $\text{cl int}(\lambda) \leq \text{cl}(\lambda)$. Thus, $\text{cl int}(\lambda) = \text{cl}(\lambda)$, in (X, T) .

- (iv). Since (X, T) is an fuzzy semi- P-space, for the fuzzy G_δ -set λ in (X, T) , $\lambda \leq \text{cl int}(\lambda)$. Now $\text{cl int}(\lambda) \leq \text{int cl}(\lambda)$, implies that $\lambda \leq \text{cl int cl}(\lambda)$, in (X, T) and hence λ is an fuzzy β -open set in (X, T) .

Proposition 3.2: If λ is an fuzzy dense and fuzzy G_δ -set in the fuzzy semi-P-space (X, T) , then $\text{cl int}(\lambda) = 1$, in (X, T) .

Proof: Let λ be an fuzzy dense and fuzzy G_δ -set in (X, T) . Since (X, T) is an fuzzy semi-P-space, the fuzzy G_δ -set λ is an fuzzy semi-open set in (X, T) . Then, $\lambda \leq \text{cl int}(\lambda)$, in (X, T) . This implies that $\text{cl}(\lambda) \leq \text{cl}[\text{cl int}(\lambda)]$. Thus, $1 \leq \text{cl int}(\lambda)$. That is, $\text{cl int}(\lambda) = 1$, in (X, T) .

Proposition 3.3: If (X, T) is an fuzzy semi-P-space, then (X, T) is an fuzzy almost P-space.

Proof: Let λ be an fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, by the proposition 3.1(i), for the fuzzy G_δ -set λ in (X, T) , $\text{int}(\lambda) \neq 0$. Hence, for the fuzzy G_δ -set λ in (X, T) , $\text{int}(\lambda) \neq 0$, implies that (X, T) is an fuzzy almost P-space.

Proposition 3.4: If λ is an fuzzy residual set in the fuzzy semi-P-space (X, T) , then:

- (i). There exists an fuzzy semi-open set μ in (X, T) such that $\mu \leq \lambda$.
- (ii). $s\text{-int}(\lambda) \neq 0$, in (X, T) .

Proof:

- (i). Let λ be an fuzzy residual set in (X, T) . Since (X, T) is an fuzzy semi-P-space, by the proposition 3.3, (X, T) is an fuzzy almost P-space. Then, by the theorem 2.2, for the fuzzy residual set λ in (X, T) , there exists an fuzzy G_δ -set μ in (X, T) such that $\mu \leq \lambda$. Since (X, T) is an fuzzy semi-P-space, the fuzzy G_δ -set μ in (X, T) , is an fuzzy semi-open in (X, T) . Thus, there exists an fuzzy semi-open set μ in (X, T) such that $\mu \leq \lambda$.
- (ii). From (i), for the fuzzy residual set λ in (X, T) , there exists an fuzzy semi-open set μ in (X, T) such that $\mu \leq \lambda$ and hence $s\text{-int}(\lambda) \neq 0$, in (X, T) .

Proposition 3.5: If λ is an fuzzy σ -nowhere dense set in the fuzzy semi-P-space (X, T) , then λ is the fuzzy semi-closed set in (X, T) .

Proof: Let λ be an fuzzy σ -nowhere dense set in (X, T) . Then, λ is the fuzzy F_σ -set in (X, T) such that $\text{int}(\lambda) = 0$. Then, $1 - \lambda$ is the fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_δ -set $(1 - \lambda)$ is the fuzzy semi-open set in (X, T) and thus λ is the fuzzy semi-closed set in (X, T) .

Proposition 3.6: If λ is an fuzzy σ -nowhere dense set in the fuzzy semi-P-space (X, T) , then

- (i). There exists an fuzzy semi-closed η in (X, T) such that $\lambda \leq \eta$.
- (ii). $s\text{-cl}(\lambda) \neq 1$, in (X, T) .

Proof:

- (i). Let λ be an fuzzy σ -nowhere dense set in (X, T) . Then, λ is the fuzzy F_σ -set with $\text{int}(\lambda) = 0$, in (X, T) and thus $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $1 - \lambda_i \in T$, for $i \in I$. Now $\text{int}(\lambda) = 0$, implies that $\text{int}(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$, in (X, T) . But

$\bigvee_{i=1}^{\infty} \text{int}(\lambda_i) \leq \text{int}(\bigvee_{i=1}^{\infty} \lambda_i)$ and thus $\bigvee_{i=1}^{\infty} \text{int}(\lambda_i) = 0$. This implies that $\text{int}(\lambda_i) = 0$. Since (λ_i) 's are fuzzy closed sets in (X, T) , $\text{int} \text{cl}(\lambda_i) = \text{int}(\lambda_i) = 0$ and then (λ_i) 's are fuzzy nowhere dense sets in (X, T) . Thus, $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$, where (λ_i) 's are fuzzy nowhere dense sets in (X, T) , implies that λ is the fuzzy first category set in (X, T) and thus $1 - \lambda$ is a fuzzy residual set in (X, T) . Since (X, T) is the fuzzy semi-P-space, by the proposition 3.4 (i), there exists a fuzzy semi-open set μ in (X, T) such that $\mu \leq 1 - \lambda$. Then, $\lambda \leq 1 - \mu$, in (X, T) . Since μ is a fuzzy semi-open set in (X, T) , $1 - \mu$ is a fuzzy semi-closed set in (X, T) . Let $\eta = 1 - \mu$. Thus, there exists a fuzzy semi-closed set η in (X, T) such that $\lambda \leq \eta$.

- (ii). From (i), for the fuzzy σ -nowhere dense set λ in (X, T) , there exists a fuzzy semi-closed set η in (X, T) such that $\lambda \leq \eta$ and hence $s\text{-cl}(\lambda) \neq 1$, in (X, T) .

Proposition 3.7: If λ is a fuzzy G_δ -set in the fuzzy semi-P-space (X, T) , then there exist a fuzzy regular closed set η in (X, T) such that $\eta \leq \text{cl}(\lambda)$.

Proof: Let λ be a fuzzy G_δ -set in (X, T) . Since (X, T) is a fuzzy semi-P-space, by the proposition 3.1 (ii), the fuzzy G_δ -set λ is a fuzzy somewhere dense set in (X, T) . Then, by the theorem 2.3, there exist a fuzzy regular closed set η in (X, T) such that $\eta \leq \text{cl}(\lambda)$.

Remarks 3.1: In view of the propositions 3.1(iii) and 3.7, one will have the following result: "If λ is a fuzzy G_δ -set in the fuzzy semi-P-space (X, T) , then there exists a fuzzy regular closed set η in (X, T) $\eta \leq \text{cl} \text{int}(\lambda)$, in (X, T) ".

Proposition 3.8: If λ is a fuzzy F_σ -set in the fuzzy semi-P-space (X, T) , then

- (i). λ is a fuzzy cs dense set in (X, T) .
(ii). $\text{cl}(\lambda) \neq 1$ in (X, T) .

Proof:

- (i). Let λ be a fuzzy F_σ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, by the proposition 3.3, (X, T) is the fuzzy almost P-space. Now $1 - \lambda$ is the fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy almost P-space, for the fuzzy G_δ -set $1 - \lambda$, $\text{int}(1 - \lambda) \neq 0$, in (X, T) . Then, $\text{int}(1 - \lambda) \leq \text{int} \text{cl}(1 - \lambda)$, implies that $\text{int} \text{cl}(1 - \lambda) \neq 0$, in (X, T) and thus $1 - \lambda$ is the fuzzy somewhere dense set in (X, T) . Hence λ is a fuzzy cs dense set in (X, T) .
(ii). Let λ be a fuzzy F_σ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, by the proposition 3.3, (X, T) is the fuzzy almost P-space. Since (X, T) is the fuzzy almost P-space, for the fuzzy G_δ -set $1 - \lambda$, $\text{int}(1 - \lambda) \neq 0$, in (X, T) . By the lemma 2.1, $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda) \neq 0$ in (X, T) . Hence $\text{cl}(\lambda) \neq 1$, in (X, T) .

Remarks 3.2: In view of the proposition 3.8 (i), one will have the following result: "Fuzzy F_σ -sets in fuzzy semi-P-spaces are fuzzy cs dense sets but not fuzzy dense sets".

Proposition 3.9: If λ is a fuzzy σ -boundary set in the fuzzy topological space (X, T) , then

- (i). λ is a fuzzy cs dense set in (X, T) .

- (ii). $\text{cl}(\lambda) \neq 1$ in (X, T) .

Proof: The proof follows from the proposition 3.7 and the theorem 2.7.

4. Fuzzy Semi-P-spaces and Other Fuzzy Topological Spaces

Proposition 4.1: If λ is a fuzzy G_δ -set in the fuzzy hyperconnected and fuzzy semi-P-space (X, T) , then

- (i). λ is a fuzzy dense set in (X, T) .
(ii). $\text{int}(1 - \lambda) = 0$, in (X, T) .

Proof:

- (i). Let λ be a fuzzy G_δ -set in (X, T) . Since (X, T) is a fuzzy semi-P-space, by the proposition 3.1(ii), the fuzzy G_δ -set λ is a fuzzy somewhere dense set in (X, T) . Also since (X, T) is the fuzzy hyperconnected space, by the theorem 2.4, the fuzzy somewhere dense set λ is a fuzzy dense set in (X, T) .
(ii). From (i), for the fuzzy G_δ -set λ in (X, T) , $\text{cl}(\lambda) = 1$ and then by the lemma 2.1, $\text{int}(1 - \lambda) = 1 - \text{cl}(\lambda) = 1 - 1 = 0$, in (X, T) .

The following proposition shows that fuzzy F_σ -sets in fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy σ -nowhere dense sets.

Proposition 4.2: If λ is a fuzzy F_σ -set in the fuzzy hyperconnected and fuzzy semi-P-space (X, T) , then λ is a fuzzy σ -nowhere dense set in (X, T) .

Proof: Let λ be a fuzzy F_σ -set in the fuzzy hyperconnected and fuzzy semi-P-space (X, T) . Now $1 - \lambda$ is the fuzzy G_δ -set in (X, T) . Then, by the proposition 4.1(ii), $\text{int}(1 - [1 - \lambda]) = 0$, in (X, T) and then $\text{int}(\lambda) = 0$, in (X, T) . Thus, λ is a fuzzy F_σ -set, in (X, T) such that $\text{int}(\lambda) = 0$. Hence λ is the fuzzy σ -nowhere dense set in (X, T) .

Remark 4.1: In view of the above propositions one will have the following results:

"Fuzzy G_δ -sets are fuzzy dense sets and fuzzy F_σ -sets are fuzzy σ -nowhere dense sets in fuzzy hyperconnected and fuzzy semi-P-spaces".

The following proposition gives the condition for the fuzzy hyperconnected and fuzzy semi-P-spaces to become fuzzy σ -Baire spaces.

Proposition 4.3: If $\text{int}[\bigvee_{i=1}^{\infty} (\lambda_i)] = 0$, where (λ_i) 's are fuzzy F_σ -sets in the fuzzy hyperconnected and fuzzy semi-P-space (X, T) , then (X, T) is a fuzzy σ -Baire space.

Proof: Let (λ_i) 's ($i = 1$ to ∞) be the fuzzy F_σ -sets in (X, T) . Since (X, T) is the fuzzy hyperconnected and fuzzy semi-P-space, by the proposition 4.2, (λ_i) 's are fuzzy σ -nowhere dense sets in (X, T) . Then, from the hypothesis, $\text{int}[\bigvee_{i=1}^{\infty} (\lambda_i)] = 0$, where (λ_i) 's are fuzzy σ -nowhere dense sets in (X, T) . This implies that (X, T) is a fuzzy σ -Baire space.

Proposition 4.4: If λ is a fuzzy residual set in the fuzzy globally disconnected and fuzzy semi-P-space (X, T) , then λ is a fuzzy semi-open set in (X, T) .

Proof: Let λ be a fuzzy residual set in (X, T) . Since (X, T) is a fuzzy globally disconnected space, by the theorem 2.5, the fuzzy residual set λ is a fuzzy G_δ -set in (X, T) . Since

(X, T) is an fuzzy semi-P-space, the fuzzy G_δ -set λ is an fuzzy semi-open set in (X, T)

Remark 4.2: In view of the above proposition, one will have the following result: "Fuzzy residual sets in fuzzy globally disconnected and fuzzy semi-P-spaces, are fuzzy

semi-open sets."

Remark 4.3: The inter-relations between fuzzy P-spaces, fuzzy semi-P-spaces and fuzzy almost P-spaces can be summarized as follows:



Figure 1. Inter relations between fuzzy Semi-P-spaces and other fuzzy P-spaces.

It is to be noted that none of the above relations are reversible. For, consider the following example:

Example 3.2: Let $X = \{a, b, c, d\}$. Let $I = [0, 1]$. The fuzzy sets α, β and γ are defined on X as follows:

$\alpha: X \rightarrow I$ is defined by $\alpha(a) = 0.6; \alpha(b) = 0.5; \alpha(c) = 0.4,$

Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \beta \vee [\alpha \wedge \gamma], \gamma \vee [\alpha \wedge \beta], \alpha \wedge [\beta \vee \gamma], \beta \wedge [\alpha \vee \gamma], \gamma \wedge [\alpha \vee \beta], \alpha \vee \beta \vee \gamma, \alpha \wedge \beta \wedge \gamma, 1\}$ is clearly the fuzzy topology on X .

On computation, one can find that $\delta = [\alpha \vee \beta] \wedge [\alpha \vee \gamma] \wedge [\beta \vee \gamma] \wedge (\alpha \vee [\beta \wedge \gamma]) \wedge (\beta \vee [\alpha \wedge \gamma]) \wedge (\gamma \vee [\alpha \wedge \beta])$ is a fuzzy G_δ -set in (X, T) . Also $cl\ int(\delta) = cl(\gamma \vee [\alpha \wedge \beta]) = 1$. Now $\delta \leq cl\ int(\delta)$, implies that the fuzzy G_δ -set δ is an fuzzy semi-open set in (X, T) . Hence (X, T) is the fuzzy semi-P-space. But (X, T) is not the fuzzy P-space, since the fuzzy G_δ -set δ is not the fuzzy open set in (X, T) .

Example 3.3: Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy

Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \beta \vee [\alpha \wedge \gamma], \gamma \vee [\alpha \wedge \beta], \alpha \wedge [\beta \vee \gamma], \beta \wedge [\alpha \vee \gamma], \gamma \wedge [\alpha \vee \beta], \alpha \vee \beta \vee \gamma, \alpha \wedge \beta \wedge \gamma, 1\}$ is the fuzzy topology on X .

On computation one can find that

$$\delta = (\alpha \vee \beta) \wedge (\alpha \vee \gamma) \wedge (\beta \vee \gamma) \wedge (\alpha \vee [\beta \wedge \gamma])$$

$$\wedge (\beta \vee [\alpha \wedge \gamma]) \wedge (\gamma \vee [\alpha \wedge \beta]) \text{ and}$$

$$\alpha \wedge \beta \wedge \gamma = (\alpha \wedge \beta) \wedge (\alpha \wedge \gamma) \wedge (\beta \wedge \gamma) \wedge (\alpha \wedge [\beta \vee \gamma])$$

$$\wedge (\beta \wedge [\alpha \vee \gamma]) \wedge (\gamma \wedge [\alpha \vee \beta]).$$

Then, δ and $\alpha \wedge \beta \wedge \gamma$ are fuzzy G_δ -sets in (X, T) . Also on computation $int(\alpha \wedge \beta \wedge \gamma) = \alpha \wedge \beta \wedge \gamma \neq 0$ and $int(\delta) = (\gamma \vee [\alpha \wedge \beta]) \neq 0$. Hence (X, T) is an fuzzy almost P-space. Now $cl\ int(\delta) = cl(\gamma \vee [\alpha \wedge \beta]) = 1 - (\beta \vee [\alpha \wedge \gamma])$ and $\delta \geq cl\ int(\delta)$, implies that the fuzzy G_δ -set δ is not the fuzzy semi-open set in (X, T) . Hence (X, T) is not the fuzzy semi-P-space.

Proposition 4.5: If the fuzzy topological space (X, T) is an fuzzy P-space, then (X, T) is an fuzzy semi-P-space.

Proof: Let λ be an fuzzy G_δ -set in (X, T) . Since (X, T) is an fuzzy P-space, the fuzzy G_δ -set λ is an fuzzy open set in (X, T) . Since each fuzzy open set is an fuzzy semi-open set in fuzzy topological spaces, λ is an fuzzy semi-open set in (X, T) and hence (X, T) is an fuzzy semi-P-space.

Proposition 4.6: If (X, T) is the fuzzy globally disconnected and fuzzy semi-P-space, then (X, T) is the fuzzy P-space.

Proof: Let λ be an fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_δ -set λ is an fuzzy semi-open set in (X, T) . Since (X, T) is the fuzzy globally disconnected space, the fuzzy semi-open set λ is the fuzzy

$\alpha(d) = 0.7,$

$\beta: X \rightarrow I$ is defined by $\beta(a) = 0.5; \beta(b) = 0.7; \beta(c) = 0.6, \beta(d) = 0.4.$

$\gamma: X \rightarrow I$ is defined by $\gamma(a) = 0.7; \gamma(b) = 0.4; \gamma(c) = 0.7, \gamma(d) = 0.6.$

sets α, β, γ , and δ are defined on X as follows:

$\alpha: X \rightarrow I$ is defined by $\alpha(a) = 0.6; \alpha(b) = 0.4; \alpha(c) = 0.5,$

$\beta: X \rightarrow I$ is defined by $\beta(a) = 0.4; \beta(b) = 0.5; \beta(c) = 0.6,$

$\gamma: X \rightarrow I$ is defined by $\gamma(a) = 0.5; \gamma(b) = 0.7; \gamma(c) = 0.4,$

$\delta: X \rightarrow I$ is defined by $\delta(a) = 0.5; \delta(b) = 0.5; \delta(c) = 0.5,$

open set in (X, T) . Hence the fuzzy G_δ -set λ is an fuzzy open set in (X, T) , implies that (X, T) is the fuzzy P-space.

The following proposition gives the condition for the fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy semi-P-spaces to become fuzzy Baire spaces

Proposition 4.7: If $cl[\bigwedge_{i=1}^{\infty}(\lambda_i)] = 1$, where (λ_i) 's are fuzzy G_δ -sets in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy semi-P-space (X, T) , then (X, T) is an fuzzy Baire space.

Proof: Suppose that (λ_i) 's are fuzzy G_δ -sets in (X, T) . Since (X, T) is the fuzzy semi-P-space, by the proposition 3.1 (ii), (λ_i) 's are fuzzy somewhere dense sets in (X, T) . Since (X, T) is the fuzzy hyperconnected space, by the Theorem 2.4, $cl(\lambda_i) = 1$ and then $1 - cl(\lambda_i) = 0$. By the lemma 2.1, $int(1 - \lambda_i) = 0$, in (X, T) . Since (X, T) is the fuzzy open hereditarily irresolvable space, $int(1 - \lambda_i) = 0$, implies that $int\ cl(1 - \lambda_i) = 0$ and thus $[1 - (\lambda_i)]$'s are fuzzy nowhere dense sets in (X, T) . Now the hypothesis $cl[\bigwedge_{i=1}^{\infty}(\lambda_i)] = 1$, implies that $1 - cl[\bigwedge_{i=1}^{\infty}(\lambda_i)] = 0$, in (X, T) and then by the lemma 2.1, that $int[1 - \bigwedge_{i=1}^{\infty}(\lambda_i)] = 0$. Thus, $int[\bigvee_{i=1}^{\infty}(1 - \lambda_i)] = 0$, where $[1 - (\lambda_i)]$'s are fuzzy nowhere dense sets in (X, T) . Hence (X, T) is the fuzzy Baire space.

Proposition 4.8: If $cl[\bigwedge_{i=1}^{\infty}(\lambda_i)] = 1$, where (λ_i) 's are fuzzy G_δ -sets in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy semi-P-space (X, T) , then (X, T) is an fuzzy second category space.

Proof: The proof follows from the proposition 4.7 and the theorem 2.6.

Proposition 4.9: If λ is an fuzzy residual set in the fuzzy submaximal and fuzzy semi-P-space (X, T) , then λ is an fuzzy semi-open set in (X, T) .

Proof: Let λ be an fuzzy residual set in (X, T) . Then, $1 - \lambda$ is the fuzzy first category set in (X, T) and thus $1 - \lambda = \bigvee_{i=1}^{\infty} (\mu_i)$, where (μ_i) 's are fuzzy nowhere dense sets in (X, T) . Since (μ_i) 's are fuzzy nowhere dense sets in (X, T) , $\text{int } cl(\mu_i) = 0$, in (X, T) . Now $\text{int}(\mu_i) \leq \text{int } cl(\mu_i)$, implies that $\text{int}(\mu_i) = 0$. By the lemma 2.1, $cl(1 - \mu_i) = 1 - \text{int}(\mu_i) = 1 - 0 = 1$ and hence $(1 - \mu_i)$'s are fuzzy dense sets in (X, T) . Since (X, T) is the fuzzy submaximal space, $(1 - \mu_i)$'s are fuzzy open sets in (X, T) . Thus (μ_i) 's are fuzzy closed sets in (X, T) . Now $1 - \lambda = \bigvee_{i=1}^{\infty} (\mu_i)$, where (μ_i) 's are fuzzy closed sets in (X, T) , implies that $1 - \lambda$ is the fuzzy F_{σ} -set in (X, T) . Hence λ is the fuzzy G_{δ} -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_{δ} -set λ is an fuzzy semi-open set in (X, T) .

Proposition 4.10: If λ is an fuzzy σ -boundary set in the fuzzy submaximal and fuzzy semi-P-space (X, T) , then λ is an fuzzy cs dense set in (X, T) .

Proof: Let λ be an fuzzy σ -boundary set in (X, T) . Since (X, T) is the fuzzy submaximal space, by the theorem 2.7, λ is an fuzzy F_{σ} -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, by the proposition 3.8(i), the fuzzy F_{σ} -set λ is an fuzzy cs dense set in (X, T) .

Proposition 4.11: If λ is an fuzzy G_{δ} -set in the fuzzy perfectly disconnected and fuzzy semi-P-space (X, T) , then $cl(\lambda)$ is an fuzzy open set in (X, T) .

Proof: Let λ be an fuzzy G_{δ} -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_{δ} -set λ is an fuzzy semi-open set in (X, T) and then $\lambda \leq cl \text{ int}(\lambda)$, in (X, T) . This implies that $\lambda \leq 1 - [1 - cl \text{ int}(\lambda)]$. Since (X, T) is the fuzzy perfectly disconnected space, for the fuzzy sets λ and $[1 - cl \text{ int}(\lambda)]$ with $\lambda \leq 1 - [1 - cl \text{ int}(\lambda)]$, $cl(\lambda) \leq 1 - cl [1 - cl \text{ int}(\lambda)]$ and then $cl(\lambda) \leq 1 - [1 - \text{int } cl \text{ int}(\lambda)]$. Thus, $cl(\lambda) \leq \text{int } cl \text{ int}(\lambda) \leq \text{int } cl(\lambda)$. But $\text{int } cl(\lambda) \leq cl(\lambda)$. Thus, $\text{int } cl(\lambda) = cl(\lambda)$. Hence $cl(\lambda)$ is an fuzzy open set in (X, T) .

Proposition 4.12: If λ is an fuzzy σ -nowhere dense set in the fuzzy perfectly disconnected and fuzzy semi-P-space (X, T) , then there exists an fuzzy semi-closed set γ in (X, T) such that $\lambda \leq \text{int}(\gamma)$.

Proof: Let λ be an fuzzy σ -nowhere dense set in (X, T) . Since (X, T) is the fuzzy perfectly disconnected by the theorem 2.8, there exists an fuzzy G_{δ} -set η in (X, T) such that $\lambda \leq \text{int}(1 - \eta)$. Since (X, T) is the fuzzy semi-P-space, the fuzzy G_{δ} -set η is an fuzzy semi-open set in (X, T) and hence $1 - \eta$ is the fuzzy semi-closed in (X, T) . Let $\gamma = 1 - \eta$. Thus, for the fuzzy σ -nowhere dense set λ in (X, T) , there exists an fuzzy semi-closed set γ in (X, T) such that $\lambda \leq \text{int}(\gamma)$.

The following proposition gives the condition for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces.

Proposition 4.13: If each fuzzy dense set is an fuzzy G_{δ} -set in the fuzzy semi-P-space (X, T) , then (X, T) is the fuzzy strongly irresolvable space.

Proof: Let λ be an fuzzy dense set in (X, T) . That is, $cl(\lambda)$

$= 1$, in (X, T) . By the hypothesis, λ is an fuzzy G_{δ} -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_{δ} -set λ is an fuzzy semi-open set in (X, T) and thus $\lambda \leq cl \text{ int}(\lambda)$, in (X, T) . This implies that $cl(\lambda) \leq cl \text{ int}(\lambda)$. Then, $1 \leq cl \text{ int}(\lambda)$. That is, $cl \text{ int}(\lambda) = 1$, in (X, T) . Hence, for the fuzzy dense set λ in (X, T) , $cl \text{ int}(\lambda) = 1$, in (X, T) , implies that (X, T) is the fuzzy strongly irresolvable space.

Proposition 4.14: If λ is an fuzzy co- σ -boundary set in the fuzzy perfectly disconnected and fuzzy semi-P-space (X, T) , then there exists an fuzzy semi-open set δ in (X, T) such that $\lambda \leq \delta$.

Proof: Let λ be an fuzzy co- σ -boundary set in (X, T) . Since (X, T) is the fuzzy perfectly disconnected by the theorem 2.9, there exists an fuzzy G_{δ} -set δ in (X, T) such that $\lambda \leq \delta$. Since (X, T) is the fuzzy semi-P-space, the fuzzy G_{δ} -set δ is an fuzzy semi-open set in (X, T) . Hence for the fuzzy co- σ -boundary set λ in (X, T) , there exists an fuzzy semi-open set δ in (X, T) such that $\lambda \leq \delta$.

Proposition 4.15: If λ is an fuzzy G_{δ} -set in the fuzzy perfectly disconnected and fuzzy semi-P-space (X, T) , then λ is an fuzzy pre-open set in (X, T) .

Proof: Let λ be an fuzzy G_{δ} -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_{δ} -set λ is an fuzzy semi-open set in (X, T) and then $1 - \lambda$ is the fuzzy semi-closed set in (X, T) . Since (X, T) is the fuzzy perfectly disconnected space, by the theorem 2.10, $1 - \lambda$ is the fuzzy pre-closed set in (X, T) and hence λ is an fuzzy pre-open set in (X, T) .

Proposition 4.16: If λ is an fuzzy F_{σ} -set in the fuzzy hyperconnected and fuzzy semi-P-space (X, T) , then $\text{int}(\lambda) = 0$, in (X, T) .

Proof: Let λ be an fuzzy F_{σ} -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, by the proposition 3.8 (i), λ is an fuzzy cs dense set in (X, T) . Then, $1 - \lambda$ is an fuzzy somewhere dense set in (X, T) . Since (X, T) is the fuzzy hyperconnected space, by the theorem 2.4, $1 - \lambda$ is the fuzzy dense set in (X, T) and then $cl(1 - \lambda) = 1$. This implies, by the lemma 2.1, that $1 - \text{int}(\lambda) = 1$ and then $\text{int}(\lambda) = 0$, in (X, T) .

Proposition 4.17: If λ is an fuzzy F_{σ} -set in the fuzzy hyperconnected and fuzzy semi-P-space (X, T) , then λ is an fuzzy σ -nowhere dense set in (X, T) .

Proof: Let λ be an fuzzy F_{σ} -set in (X, T) . Since (X, T) is the fuzzy hyper-connected and fuzzy semi-P-space, by the proposition 4.16, $\text{int}(\lambda) = 0$, in (X, T) . Hence λ is an fuzzy F_{σ} -set in (X, T) such that $\text{int}(\lambda) = 0$ and hence λ is an fuzzy σ -nowhere dense set in (X, T) .

Remark 4.4: In view of the above proposition, one will have the following result: "Fuzzy F_{σ} -sets in fuzzy hyperconnected and fuzzy semi-P-spaces are fuzzy σ -nowhere dense sets".

The following proposition shows that fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy irresolvable spaces.

Proposition 4.18: If (X, T) is the fuzzy hyperconnected and fuzzy semi-P-space, then (X, T) is the fuzzy irresolvable space.

Proof: Let λ be an fuzzy F_{σ} -set in (X, T) . Since (X, T) is the fuzzy hyperconnected and fuzzy semi-P-space, by the proposition 4.16, $\text{int}(\lambda) = 0$, in (X, T) . Then, $cl(1 - \lambda) = 1 - \text{int}(\lambda) = 1$. Since (X, T) is the fuzzy semi-P-space, by

the proposition 3.8 (ii), $cl(\lambda) \neq 1$, in (X, T) . Thus, for the fuzzy dense set $1-\lambda$ in (X, T) , $cl [1-(1-\lambda)] = cl(\lambda) \neq 1$, in (X, T) . Hence (X, T) is the fuzzy irresolvable space.

Proposition 4.19: If λ is a fuzzy σ -boundary set in the fuzzy hyperconnected and fuzzy semi-P-space (X, T) , then λ is an fuzzy σ -nowhere dense set in (X, T) .

Proof: Let λ be an fuzzy σ -boundary set in (X, T) . Then, by the theorem 2.7, λ is an fuzzy F_σ -set in (X, T) . Since (X, T) is the fuzzy hyperconnected and fuzzy semi-P-spaces, by the proposition 4.16, $int(\lambda) = \emptyset$, in (X, T) . Hence λ is an fuzzy F_σ -set in (X, T) such that $int(\lambda) = \emptyset$ and hence λ is the fuzzy σ -nowhere dense set in (X, T) .

Remark 4.5: In view of the above proposition, one will have the following result: "Fuzzy σ -boundary sets in fuzzy hyperconnected and fuzzy semi-P-spaces are fuzzy σ -nowhere dense sets".

The following proposition gives the condition for fuzzy G_δ -sets in fuzzy globally disconnected and fuzzy semi-P-spaces to become fuzzy simply open sets.

Proposition 4.20: If λ is the fuzzy dense and fuzzy G_δ -set in the fuzzy globally disconnected and fuzzy semi-P-space (X, T) , then λ is the fuzzy simply open set in (X, T) .

Proof: Let λ be an fuzzy dense and fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_δ -set λ is the fuzzy semi-open set in (X, T) . Thus, λ is the fuzzy semi-open and fuzzy dense set in (X, T) . Since (X, T) is the fuzzy globally disconnected space, by the theorem 2.12, λ is the fuzzy simply open set in (X, T) .

Proposition 4.21: If λ is the fuzzy residual set in the fuzzy globally disconnected and fuzzy semi-P-space (X, T) , then $cl int(\lambda) = cl(\lambda)$, in (X, T) .

Proof: Let λ be an fuzzy residual set in (X, T) . Since (X, T) is an fuzzy globally disconnected space, by the theorem 2.5, λ is the fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, for the fuzzy G_δ -set λ in (X, T) , by the proposition 3.1(iii), $cl int(\lambda) = cl(\lambda)$, in (X, T) .

Proposition 4.22: If λ is the fuzzy F_σ -set in the fuzzy globally disconnected and fuzzy semi-P-space (X, T) , then,

- (i). $cl(1-\lambda) = 1 - cl int(\lambda)$, in (X, T) .
- (ii). $cl int(\lambda) = int(\lambda)$, in (X, T) .

Proof:

- (i). Let λ be the fuzzy F_σ -set in (X, T) . Then, $1-\lambda$ is the fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy semi-P-space, the fuzzy G_δ -set $(1-\lambda)$ is the fuzzy semi-open set in (X, T) and thus λ is the fuzzy semi-closed set in (X, T) . Since (X, T) is the fuzzy globally disconnected space, by the theorem 2.1, for the fuzzy semi-closed set λ in (X, T) , $cl(1-\lambda) = 1 - cl int(\lambda)$, in (X, T) .

- (ii). From (i), for the fuzzy F_σ -set λ in (X, T) , $cl(1-\lambda) = 1 - cl int(\lambda)$, in (X, T) . Then, $1 - int(\lambda) = 1 - cl int(\lambda)$ and thus $int(\lambda) = cl int(\lambda)$, in (X, T) .

The following proposition gives the condition for the fuzzy globally disconnected and fuzzy semi-P-spaces to become fuzzy extremally disconnected spaces.

Proposition 4.23: If each fuzzy open set is the fuzzy F_σ -set in the fuzzy globally disconnected and fuzzy semi-P-space (X, T) , then (X, T) is an fuzzy extremally disconnected

space.

Proof: Let λ be an fuzzy open set in (X, T) . By the hypothesis, λ is an fuzzy F_σ -set in the fuzzy globally disconnected and fuzzy semi-P-space (X, T) . Then, by the proposition 4.22 (ii), $cl int(\lambda) = int(\lambda)$, in (X, T) . Since λ is the fuzzy open set in (X, T) , $int(\lambda) = \lambda$ and then $cl(\lambda) = \lambda \in T$. Hence, for the fuzzy open set λ in (X, T) , $cl(\lambda) \in T$ implies that (X, T) is the fuzzy extremally disconnected space.

The following proposition gives the condition for the fuzzy GID spaces to become fuzzy semi-P-spaces.

Proposition 4.24: If each fuzzy G_δ -set is an fuzzy dense set in the fuzzy GID space (X, T) , then (X, T) is the fuzzy semi-P-space.

Proof: Let λ be an fuzzy G_δ -set in (X, T) . By the hypothesis, λ is an fuzzy dense set in (X, T) . Then, λ is an fuzzy dense and fuzzy G_δ -set in (X, T) . Since (X, T) is the fuzzy GID space, by the theorem 2.11, the fuzzy G_δ -set λ is the fuzzy semi-open set in (X, T) . This implies that (X, T) is an fuzzy semi-P-space.

Proposition 4.25: If λ is an fuzzy first category set in the fuzzy submaximal and fuzzy semi-P-space (X, T) , then λ is the fuzzy semi-closed set in (X, T) .

Proof: Let λ be an fuzzy first category set in (X, T) . Then, $1-\lambda$ is an fuzzy residual set in (X, T) . Since (X, T) is the fuzzy submaximal and fuzzy semi-P-space, by the proposition 4.9, $1-\lambda$ is the fuzzy semi-open set in (X, T) . Hence λ is the fuzzy semi-closed set in (X, T) .

Remark 4.6: In view of the above proposition, one will have the following result: "Fuzzy first category sets in fuzzy submaximal and fuzzy semi-P-spaces are fuzzy semi-closed sets".

5. Conclusion

In this paper, the notion of fuzzy semi-P-space is introduced by means of fuzzy semi-openness of fuzzy G_δ -sets. It is established that the fuzzy G_δ -sets are fuzzy somewhere dense sets, fuzzy β -open sets and fuzzy σ -nowhere dense sets are fuzzy semi-closed sets in fuzzy semi-P-spaces. It is established that the class of fuzzy semi-P-spaces lies between the classes of fuzzy P-spaces and fuzzy almost P-spaces. Also it is obtained that fuzzy F_σ -sets in fuzzy semi-P-spaces are fuzzy cs dense sets but not fuzzy dense sets and fuzzy F_σ -sets in fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy σ -nowhere dense sets. Also it is obtained that fuzzy residual sets in fuzzy globally disconnected and fuzzy semi-P-spaces, are fuzzy semi-open sets. The conditions for the fuzzy semi-P-spaces to become fuzzy P-spaces and for the fuzzy hyperconnected and fuzzy semi-P-spaces to become fuzzy σ -Baire spaces, are obtained. Also the conditions for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces and for the fuzzy hyperconnected, fuzzy open hereditarily irresolvable and fuzzy semi-P-spaces to become fuzzy Baire spaces are obtained. The conditions under which the fuzzy GID spaces become fuzzy semi-P-spaces and fuzzy globally disconnected and fuzzy semi-P-spaces become fuzzy extremally disconnected spaces, are also obtained in this paper.

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