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Numerical Representation of MHD Turbulence Prior to the Ultimate Phase of Decay

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Abstract: Following Deissler's approach the magnetic field fluctuation in MHD turbulence prior to the ultimatephase of decay is studied. Two and three point correlation equations have been obtained and the set of equations is made determinate by neglecting the quadruple correlations in comparison with second and third order correlations. The correlation equations are changed to spectral form by taking their Fourier transforms. The decay law for magnetic field fluctuations is obtained and discussed the problem numerically and represented the results graphically.

Keywords: Correlation Function, Deissler's Method, Fourier-Transformation, Matlab, Navier-Stokes Equation

1. Introduction

Magneto hydrodynamic (MHD) turbulence is characterized by nonlinear interactions among fluctuations of the magnetic field and flow velocity over a range of spatial and temporal scales. It plays an important role in the transport of energetic particles. Magneto hydrodynamics turbulence has been employed as a physical model for a wide range of applications in astrophysical and space plasma physics. The fundamental aspects of MHD turbulence include spectral energy transfer, non-locality, and anisotropy, each of which is related to the multiplicity of dynamical time scales that may be present. These basic issues can be discussed based on the concepts of magnetic Prandtle number of the small scales in the magnetic field. The magnetic Prandtle number defined as the ratio between the kinematic viscosity and the magnetic diffusivity. Boyd (2001) discussed Chebyshev and Forier spectral methods. Shebalin (2002) explained the statistical mechanics of ideal homogeneous turbulence. Biskamp (2003) obtained magneto hydrodynamic turbulence. Islam and Sarker (2001) developed the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Shebalin (2006) also oriented ideal homogeneous magnetohydrodynamic turbulence in the presence of rotation and a mean magnetic field. Deissler (1958, 1960) developed a theory 'on the decay of homogeneous turbulence for times before the final period.' By considering Deissler's theory, Loeffler and Deissler (1961) studied the decay of temperature fluctuation in homogeneous turbulence before the final period. Bkar Pk *et al.* (2013) illustrated the decay of MHD turbulence prior to the ultimate phase in presence of dust particle for four-point correlation. Chandrasekhar (1951) obtained the invariant theory of isotropic turbulence in magneto-hydrodynamics. Rahaman (2010) obtained the decay of first order reactant in incompressible MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Corrsin (1951) considered the spectrum of isotropic temperature fluctuations in isotropic turbulence. In their approach they considered the two- and three-point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms analytically. Here, two- and three-point correlation equations have been considered, and the same approach of Deissler (1960) is applied to a theory of decaying homogeneous turblence. Sarker and Kishore (1991) derived the problem decay of the MHD turbulence before the final period analytically. In this chapter, we have discussed the problem numerically and represented the results graphically. We have shown that if the magnetic diffusivity is constant and kinematic viscosity is transferable then the fluctuation of the decay curves is linear

and is parallel to the direction of x-axis and the decay is very small at constant time.

2. Mathematical Formulation

For two points, we need two equations. Let the induction equation of magnetic field at the point P is

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = (\frac{\vartheta}{P_M}) \frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad (1)$$

and at the point P' will be

$$\frac{\partial h_j{}'}{\partial t} + u_k \frac{\partial h_j{}'}{\partial x_k{}'} - h_k \frac{\partial u_j{}'}{\partial x_k{}'} = (\frac{\vartheta}{P_M}) \frac{\partial^2 h_j{}'}{\partial x_k{}' \partial x_k{}'} \quad (2)$$

where

$W = \frac{P}{\rho} + \frac{1}{2}\left|\vec{h}\right|^2$ is the total MHD pressure,

$p(x,t)$ is the hydrodynamic pressure,

ρ is the fluid density,

$P_M = \frac{\vartheta}{\lambda}$ is defined as the magnetic prandtle number,

ϑ is the kinematic viscosity,

λ is the magnetic diffusivity,

$h_i(x,t)$ is the magnetic field fluctuation,

$u_i(x,t)$ is the turbulent velocity,

t is the time, x_k is the space coordinate, and the repeated subscripts are summed from 1 to 3.

Multiplying the equation (1) by $h_j{}'$ and (2) by h_i we get respectively

$$h_j{}' \frac{\partial h_i}{\partial t} + h_j{}' u_k \frac{\partial h_i}{\partial x_k} - h_j{}' h_k \frac{\partial u_i}{\partial x_k} = (\frac{\vartheta}{P_M}) h_j{}' \frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad (3)$$

$$h_i \frac{\partial h_j{}'}{\partial t} + h_i u_k \frac{\partial h_j{}'}{\partial x_k{}'} - h_i h_k \frac{\partial u_j{}'}{\partial x_k{}'} = (\frac{\vartheta}{P_M}) h_i \frac{\partial^2 h_j{}'}{\partial x_k{}' \partial x_k{}'} \quad (4)$$

Adding equation (3) and (4) and taking ensemble average with transformations

$$\frac{\partial}{\partial r_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x_k{}'}$$

and Chandrasekhar's relation (1951)

$$\langle u_k h_i h_j{}' \rangle = -\langle h_i u_k{}' h_j{}' \rangle and \langle h_i u_j{}' h_k{}' \rangle = -\langle u_i h_k h_j{}' \rangle$$

we obtain

$$\frac{\partial}{\partial t}\langle h_i h_j{}' \rangle + 2\left[\frac{\partial}{\partial r_k}\langle u_k{}' h_i h_j{}' \rangle - \frac{\partial}{\partial r_k}\langle u_i h_k h_j{}' \rangle\right] = 2\frac{\vartheta}{P_M}\frac{\partial^2}{\partial r_k \partial r_k}\langle h_i h_j{}' \rangle \quad (5)$$

We have three dimensional Fourier transforms

$$\langle h_i h_j{}' \rangle(\vec{r}) = \int_{-\infty}^{\infty} \langle \Psi_i \Psi_j{}' \rangle(\vec{K}) \exp(i\vec{K}.\vec{r})\, d\vec{K} \quad (6)$$

$$\langle u_i h_k h_j{}' \rangle(\vec{r}) = \int_{-\infty}^{\infty} \langle \alpha_i \Psi_k \Psi_j{}' \rangle(\vec{K}) \exp(i\vec{K}.\vec{r})\, d\vec{K} \quad (7)$$

Integrating the subscripts i and j and then integrating the points P and P', we have

$$\langle u_k h_i h_j{}' \rangle(\vec{r}) = \langle u_k h_i h_j{}' \rangle(-\vec{r}) = \int_{-\infty}^{\infty} \langle \alpha_k \Psi_i \Psi_j{}' \rangle(-\vec{K}) \exp(i\vec{K}.\vec{r})\, d\vec{K} \quad (8)$$

Putting these three equations in (5) becomes

$$\frac{\partial}{\partial t}\langle \Psi_i \Psi_j{}' \rangle(\vec{K}) + 2\frac{\vartheta}{P_M} k^2 \langle \Psi_i \Psi_j{}' \rangle(\vec{K}) = 2ik_k[\langle \alpha_i \Psi_k \Psi_j{}' \rangle(\vec{K}) - \langle \alpha_k \Psi_i \Psi_j{}' \rangle(-\vec{K})]$$

The above tensor equation becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}\langle \Psi_i \Psi_i{}' \rangle(\vec{K}) + 2\frac{\vartheta}{P_M} k^2 \langle \Psi_i \Psi_i{}' \rangle(\vec{K}) = 2ik_k\left[\langle \alpha_i \Psi_k \Psi_i{}' \rangle(\vec{K}) - \langle \alpha_k \Psi_i \Psi_i{}' \rangle(-\vec{K})\right] \quad (9)$$

The term on the right hand side of the equation (9) is called energy transfer term while the second term on the left hand side is the dissipation term.

We consider the momentum equation of MHD turbulence at the point P, and the induction equations of magnetic field fluctuation at P' and P'' as follows

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial W}{\partial x_l} + \vartheta \frac{\partial^2 u_l}{\partial x_k \partial x_k} \quad (10)$$

$$\frac{\partial h_i{}'}{\partial t} + u_k{}' \frac{\partial h_i{}'}{\partial x_k{}'} - h_k{}' \frac{\partial u_i{}'}{\partial x_k{}'} = \frac{\vartheta}{P_M} \frac{\partial^2 h_i{}'}{\partial x_k{}' \partial x_k{}'} \quad (11)$$

$$\frac{\partial h_j{}''}{\partial t} + u_k{}'' \frac{\partial h_j{}''}{\partial x_k{}''} - h_k{}'' \frac{\partial u_j{}''}{\partial x_k{}''} = \frac{\vartheta}{P_M} \frac{\partial^2 h_j{}''}{\partial x_k{}'' \partial x_k{}''} \quad (12)$$

Multiplying equation (10) by $h_i h_j{}''$, (11) by $u_i h_j{}''$ and (12) by $u_i h_i{}'$, we obtain

$$h_i h_j{}'' \frac{\partial u_l}{\partial t} + u_k h_i h_j{}'' \frac{\partial u_l}{\partial x_k} - h_k h_i h_j{}'' \frac{\partial h_l}{\partial x_k} = -h_i h_j{}'' \frac{\partial W}{\partial x_l} + \vartheta h_i h_j{}'' \frac{\partial^2 u_l}{\partial x_k \partial x_k} \quad (13)$$

$$u_i h_j{}'' \frac{\partial h_i{}'}{\partial t} + u_k{}' u_i h_j{}'' \frac{\partial h_i{}'}{\partial x_k{}'} - h_k{}' u_i h_j{}'' \frac{\partial u_i{}'}{\partial x_k{}'} = \frac{\vartheta}{P_M} u_i h_j{}'' \frac{\partial^2 h_i{}'}{\partial x_k{}' \partial x_k{}'} \quad (14)$$

$$u_i h_i{}' \frac{\partial h_j{}''}{\partial t} + u_k{}'' u_i h_i{}' \frac{\partial h_j{}''}{\partial x_k{}''} - h_k{}'' u_i h_i{}' \frac{\partial u_j{}''}{\partial x_k{}''} = \frac{\vartheta}{P_M} u_i h_i{}' \frac{\partial^2 h_j{}''}{\partial x_k{}'' \partial x_k{}''} \quad (15)$$

Combining equations (13), (14), (15), and taking time average and using the transformation, we obtain

$$\frac{\partial}{\partial x_k{}'} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x_k{}''} = \frac{\partial}{\partial r_k{}'},$$

and

$$\frac{\partial}{\partial x_k} = -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'}\right)$$

We get

$$\frac{\partial}{\partial t}\langle u_l h_i', h_j''\rangle - \frac{\vartheta}{P_M}\Big[(1+P_M)\frac{\partial^2}{\partial r_k \partial r_k}\langle u_l h_i' h_j''\rangle + (1+P_M)\frac{\partial^2}{\partial r_k' \partial r_k'}\langle u_l h_i' h_j''\rangle + 2P_M \frac{\partial^2}{\partial r_k \partial r_k'}\langle u_l h_i' h_j''\rangle\Big] = \frac{\partial}{\partial r_k}\langle u_l u_k h_i' h_j''\rangle + \frac{\partial}{\partial r_k'}\langle u_l u_k h_i' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle - \frac{\partial}{\partial r_k}\langle u_l u_k' h_i' h_j''\rangle + \frac{\partial}{\partial r_k}\langle u_l u_i' h_k' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle u_l u_k'' h_i' h_j''\rangle + \frac{\partial}{\partial r_k'}\langle u_l u_j'' h_i' h_k''\rangle + \frac{\partial}{\partial r_l}\langle (W h_i' h_j''\rangle + \frac{\partial}{\partial r_l'}\langle W h_i' h_j''\rangle \quad (16)$$

Using Fourier transforms

$$\langle u_l h_i'(\vec{r}), h_j''(\vec{r}')\rangle =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \Phi_1 \beta_i'(\vec{K})\beta_j''(\vec{K}')\rangle . \exp[\mathrm{i}(\vec{K}.\vec{r} + \vec{K}'.\vec{r}')]\, d\vec{K} d\vec{K}' \quad (17)$$

and

$$\langle W h_i'(\vec{r}), h_j''(\vec{r}')\rangle =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i'(\vec{K})\beta_j''(\vec{K}')\rangle . \exp[\mathrm{i}(\vec{K}.\vec{r} + \vec{K}'.\vec{r}')]\, d\vec{K} d\vec{K}' \quad (18)$$

with transformation

$$\langle u_l u_k'' h_j'' h_i'\rangle = \langle u_l u_k' h_i' h_j''\rangle$$

and

$$\langle u_l u_j'' h_i' h_k''\rangle = \langle u_l u_i' h_k' h_j''\rangle$$

The equation (17) reduces into the form

$$\frac{\partial}{\partial t}\langle \Phi_l \beta_i' \beta_j''\rangle + \frac{\vartheta}{P_M}\big[(1+P_M)k^2 + (1+P_M)k'^2 + 2P_M k_k k_k'\big]\langle \Phi_l \beta_i' \beta_j''\rangle = i(k_k + k_k')\langle \Phi_l \Phi_k \beta_i' \beta_j''\rangle - i(k_k + k_k')\langle \beta_l \beta_k \beta_i' \beta_j''\rangle - i(k_k + k_k')\langle \Phi_l \Phi_k' \beta_i' \beta_j''\rangle + i(k_k + k_k')\langle \Phi_l \Phi_i' \beta_k' \beta_j''\rangle + i(k_l + k_l')\langle \gamma \beta_i' \beta_j''\rangle \quad (19)$$

Taking contraction of the indices i *and* j in equation (19), we get the spectrum equation at three point correlation as:

$$\frac{\partial}{\partial t}\langle \Phi_l \beta_i' \beta_i''\rangle + \frac{\vartheta}{P_M}\big[(1+P_M)(k^2 + k'^2) + 2P_M k k'\big]\langle \Phi_l \beta_i' \beta_i''\rangle = i(k_k + k_k')\langle \Phi_l \Phi_k \beta_i' \beta_i''\rangle - i(k_k + k_k')\langle \beta_l \beta_k \beta_i' \beta_i''\rangle - i(k_k + k_k')\langle \Phi_l \Phi_k \beta_i' \beta_i''\rangle + i(k_k + k_k')\langle \Phi_l \Phi_i' \beta_k' \beta_i''\rangle + i(k_l + k_l')\langle \gamma \beta_i' \beta_i''\rangle \quad (20)$$

Taking derivative of equation (10) at $P$with respect to x_l we get

$$-\frac{\partial^2 \mathrm{W}}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}(u_l u_k - h_l h_k) \quad (21)$$

For independent variables $r\ and\ r'$ multiplying equation (21) by $h_i' h_j''$, taking time averages we get

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + \frac{\partial^2}{\partial r_l' \partial r_l'} + 2\frac{\partial^2}{\partial r_l \partial r_l'}\right]\langle W h_i' h_j''\rangle = \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r_l' \partial r_k} + \frac{\partial^2}{\partial r_l \partial r_k'} + \frac{\partial^2}{\partial r_l' \partial r_k'}\right].(\langle u_l u_k h_i' h_j''\rangle - \langle h_l h_k h_i' h_j''\rangle) \quad (22)$$

Applying Fourier transforms of the equation (22), we get

$$-\langle \gamma \beta_i' \beta_j''\rangle = \frac{(k_l k_k + k_l' k_k + k_l k_k')(\langle \Phi_l \Phi_k \beta_i' \beta_j''\rangle - \langle \beta_l \beta_k \beta_i' \beta_j''\rangle)}{k^2 + k'^2 + 2k_l k_l'} \quad (23)$$

where $\langle \gamma \beta_i' \beta_j''\rangle$ is the pressure correlation.

3. Results and Discussion

The solution is obtained by considering the two-point correlation after neglecting the third order correlations the three point correlation equations is considered and the quadruple correlations are neglected. The terms $\langle \gamma \beta_i' \beta_j''\rangle$ associated with the pressure correlations must be neglected. Thus, neglecting all the terms on the right hand side of the equation (2), we have

$$\frac{\partial}{\partial t}\langle \Phi_l \beta_i' \beta_j''\rangle + \frac{\vartheta}{P_M}\big[(1+P_M)(K^2 + K'^2) + 2P_M K K'\big]\langle \Phi_l \beta_i' \beta_j''\rangle = 0 \quad (24)$$

Integrating the equation (24) between $t_0 and\ t$ with inner multiplication by k_k and gives

$$k_k\langle \Phi_l \beta_i' \beta_j''\rangle = k_k\langle \Phi_l \beta_i' \beta_j''\rangle_0 \exp\Big[-\frac{\vartheta}{P_M}\big\{(1+P_M)(k^2 + k'^2) + 2P_M K K' \cos\theta\big\}(t_0 - t)\Big] \quad (25)$$

Where θ is the angle between $k and\ k'$.

Now, by letting $r' = 0$ in the equation (18) and comparing with the equation (7) and (8), we obtain

$$\langle \alpha_i \Psi_k \Psi_i'\rangle(\vec{K}) = \int_{-\infty}^{\infty}\langle \Phi_l \beta_i' \beta_i''\rangle \,\mathrm{d}\vec{K} \quad (26)$$

$$\langle \alpha_k \Psi_i \Psi_i'\rangle(-\vec{K}) = \int_{-\infty}^{\infty}\langle \Phi_l \beta_i'(-\vec{K})\beta_i''(-\vec{K}')\rangle \,\mathrm{d}\vec{K} \quad (27)$$

Substituting the equations (25), (26) and (27) in equation (9), we get

$$\frac{\partial}{\partial t}\langle \Psi_i \Psi_i'\rangle(\vec{K}) + 2\frac{\vartheta}{P_M}K^2\langle \Psi_i \Psi_i'\rangle(\vec{K}) = 2ik_k[\langle \Phi_l \beta_i' \beta_i''\rangle - \langle \Phi_l \beta_i'(-\vec{K})\beta_i''(-\vec{K}')\rangle]_0 . \exp\Big[-\frac{\vartheta}{P_M}\big\{(1+P_M)(k^2 + k'^2) + 2P_M k k' \cos\theta\big\}(t - t_0)]\mathrm{d}\vec{K}' \quad (28)$$

Now, $\mathrm{d}\vec{K}' = \mathrm{d}K_1' \mathrm{d}K_2' \mathrm{d}K_3'$ can be expressed in terms of $k' and \theta$.

Following Sarker and kishore (1991) and using Loeffler and Deissler (1961) and assumption and then integrating

w.r.to $\cos\theta$, we get,

$$\frac{\partial H}{\partial t}+\frac{2\vartheta k^2 H}{P_M}=G \tag{29}$$

where $H=2\pi k^2\langle\Psi_i\Psi_i'\rangle(\vec{K})$ is the magnetic energy spectrum function and G is the energy transfer term given by

$$G=-\frac{\xi_0}{\vartheta(t-t_0)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right).[\exp\{-\frac{\vartheta}{P_M}(t-t_0)\left[(1+P_M)(k^2+k'^2)-2P_Mkk'\right]\}-\exp\{-\frac{\vartheta}{P_M}(t-t_0)\left[(1+P_M)(k^2+k'^2)+2P_Mkk'\right]\}]\,dk' \tag{30}$$

Integrating equation (30) with respect to k', we have

$$G=-\frac{-\sqrt{\pi}\xi_0P_M^{\frac{5}{2}}}{\vartheta^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}(1+P_M)^{\frac{5}{2}}}\exp\left\{\frac{-\vartheta(t-t_0)}{P_M}.(\frac{1+2P_M}{1+P_M})k^2\right\}[\frac{15P_Mk^4}{4\vartheta^2(t-t_0)^2(1+P_M)}+\left\{\frac{5P_M^2}{(1+P_M)^2\vartheta(t-t_0)}-\frac{3}{2\vartheta(t-t_0)}\right\}k^6+\frac{P_M}{1+P_M}\left\{\frac{P_M{}^2}{(1+P_M)^2}-1\right\}k^8] \tag{31}$$

Solving the linear equation (29)and using Corrsin(1951) relation $J(k)=\frac{N_0k^2}{\pi}$ is a constant,we get

$$H=\frac{N_0k^2}{\pi}\exp\left[-2\frac{\vartheta k^2(t-t_0)}{P_M}\right]+\frac{\sqrt{\pi}\xi_0P_M^{\frac{5}{2}}}{\vartheta^{\frac{3}{2}}(1+P_M)^{\frac{7}{2}}}.\exp\left\{\frac{-\vartheta k^2(t-t_0)(1+2P_M)}{P_M(1+P_M)}\right\}[\frac{3P_Mk^4}{2\vartheta^2(t-t_0)^{\frac{5}{2}}}+\frac{(7P_M^2-6P_M)k^6}{3(1+P_M)(t-t_0)^{\frac{3}{2}}}-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0)^{\frac{1}{2}}}+\frac{8\sqrt{\vartheta}(3P_M^2-2P_M+3)k^9}{3(1+P_M)^{\frac{5}{2}}P_M^{\frac{1}{2}}}F(\omega)] \tag{32}$$

where

$$F(\omega)=e^{-\omega^2}\int_0^\omega e^{x^2}\,dx,\omega=k\sqrt{[\frac{\vartheta(t-t_0)}{P_M(1+P_M)}]}$$

By setting

$$\vec{r}=0,j=i,$$

$$d\vec{K}=-2\pi k^2d(\cos\theta)dk and H=2\pi k^2\langle\Psi_i\Psi_i\rangle$$

in the equation (20), we get the expression for magnetic energy decay as

$$\frac{\langle h_ih_i'\rangle}{2}=\int_0^\infty Hdk \tag{33}$$

Substituting equation (32) into (33) and after integration w. r.to time, we can write

$$\langle h^2\rangle=A(t-t_0)^{-\frac{3}{2}}+B(t-t_0)^{-5} \tag{34}$$

where

$$Q=\frac{\pi P_M^6}{(1+P_M)(1+2P_M)^{\frac{5}{2}}}[\frac{9}{16}+\frac{5P_M(7P_M-6)}{(1+2P_M)}-\frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)^2}+\cdots]$$

$$A=\frac{N_0P_M^{\frac{3}{2}}\vartheta^{-\frac{3}{2}}}{8\sqrt{2\pi}}$$

$$B=\xi_02Q\vartheta^{-6}$$

which is the decay law for magnetic energy fluctuation before the final period.

Now we are going to discuss the problem in numerical analysis:

Table 1. *If λ is fixed and ϑ varies from 0.05 to 0.15.*

ϑ	λ	P_M	Q	A	B
0.05	2	0.025	-4.96×10^{-10}	6.97×10^{-5}	-0.064
0.10	2	0.050	-7.24×10^{-8}	1.971×10^{-4}	-0.144
0.15	2	0.075	-1.08×10^{-6}	3.622×10^{-4}	-0.190

Table 2. *Different values of A and B if λ is constant.*

ϑ	λ	P_M	Q	A	B
0.20	2	0.100	-5.27×10^{-6}	5.576×10^{-4}	-0.164
0.25	2	0.125	-2.08×10^{-6}	7.791×10^{-4}	-0.170
0.30	2	0.150	-5.645×10^{-5}	1.024×10^{-3}	-0.155

Table 3. *Values of A and B for the variation of ϑ.*

ϑ	λ	P_M	Q	A	B
0.35	2	0.175	-1.262×10^{-4}	1.291×10^{-3}	-0.137
0.40	2	0.200	-2.453×10^{-4}	1.577×10^{-3}	-0.120
0.45	2	0.225	-4.293×10^{-4}	1.882×10^{-3}	-0.103

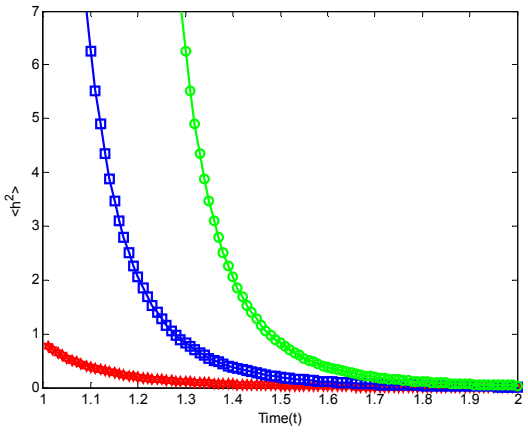

Figure 1. *Energy decay curves for $A=6.97\times10^{-5}$ and $B=0.064$, t_0=0.4, 0.7, and 0.9.*

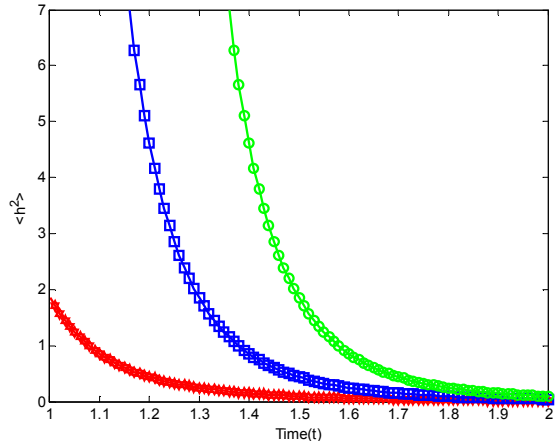

Figure 2. *Energy decay curves for* $A = 1.971 \times 10^{-4}$ *and* $B = -0.144$, t_0*=0.4, 0.7, and 0.9.*

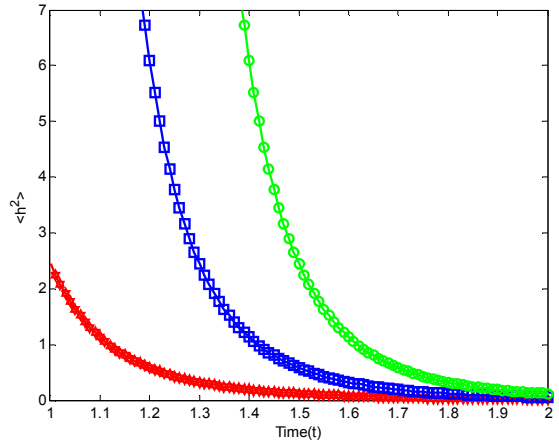

Figure 3. *Energy decay curves for* $A = 3.622 \times 10^{-4}$ *and* $B = -0.190$, t_0*=0.4, 0.7, and 0.9.*

If λ (magnetic diffusivity) is fixed and ϑ (kinematic viscosity) varies from 0.05 to 0.15 in Table 1, Magnetic Prandtle number P_M is increased for increasing of kinematic viscosity (ϑ) because they are proportional to each other.

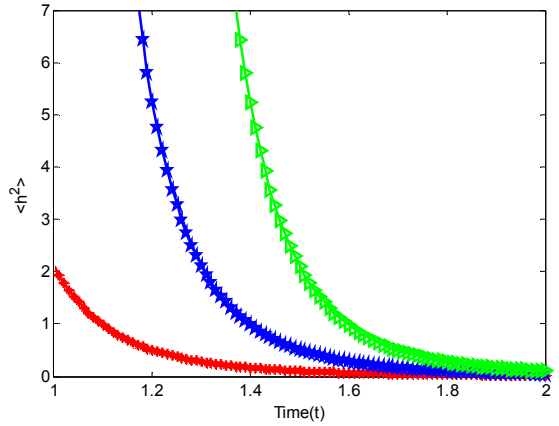

Figure 4. *Energy decay curves for* $A = 5.576 \times 10^{-4}$ *and* $B = -0.164$, t_0*=0.4, 0.7, and 0.9.*

For fixed time (t_0 =0.4, 0.7, 0.9) and for different values of A and B, the total energy $\langle h^2 \rangle$ is increasing from Figure 1 to Figure 3. Here time (t) has been taken in the direction of x-axis and total energy $\langle h^2 \rangle$ in the direction of y-axis. In the direction of x-axis for time the limit has been taken from 1 to 2 and in the direction of y-axis for total energy the limit has been taken from 0 to 7.

When λ is fixed and ϑ varies from 0.20 to 0.30, different values of A & B in Table 2, the change of total energy is very small from Figure 4 to Figure 6.

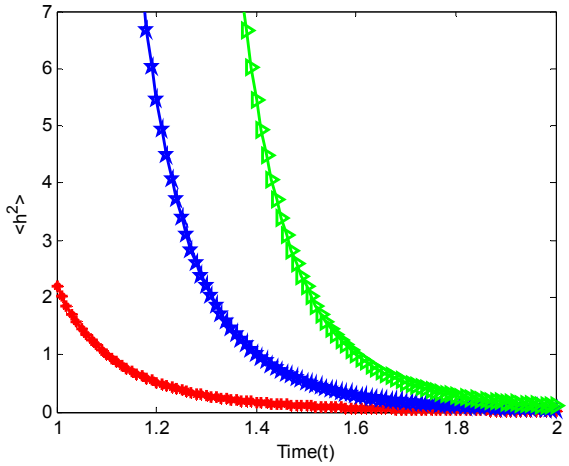

Figure 5. *Energy decay curves for* $A = 7.791 \times 10^{-4}$ *and* $B = -0.170$, t_0*=0.4, 0.7, and 0.9.*

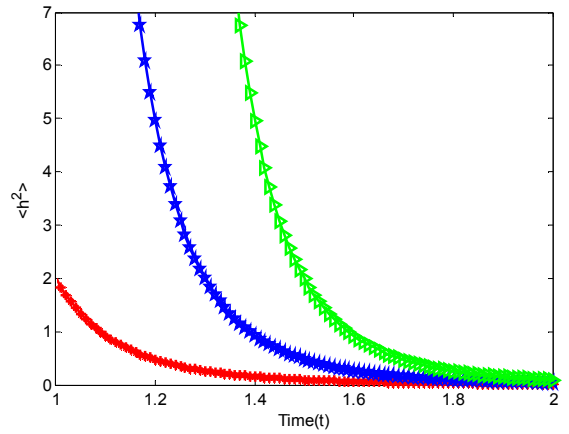

Figure 6. *Energy decay curves for* $A = 1.024 \times 10^{-3}$ *and* $B = -0.155$, t_0*=0.4, 0.7, and 0.9.*

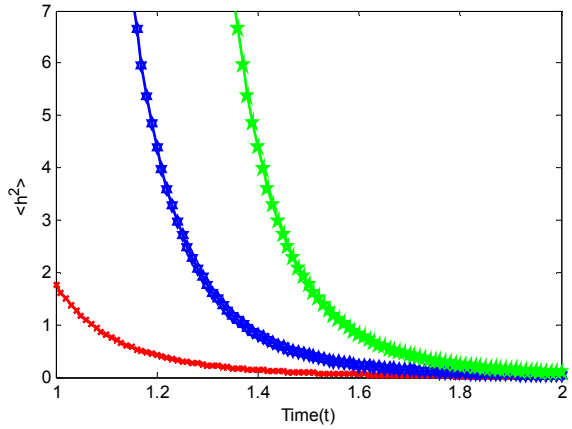

Figure 7. *Energy decay curves for* $A = 1.291 \times 10^{-3}$ *and* $B = -0.137$, t_0*=0.4, 0.7, and 0.9.*

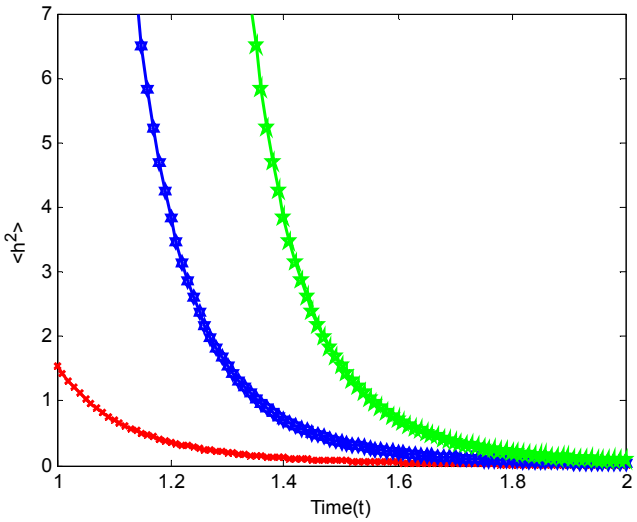

Figure 8. *Energy decay curves for* $A = 1.577 \times 10^{-3}$ *and* $B = -0.120$, t_0*=0.4, 0.7, and 0.9.*

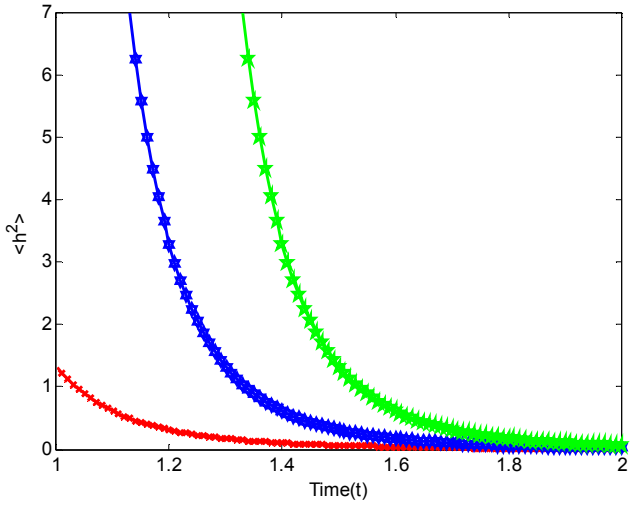

Figure 9. *Energy decay curves for* $A = 1.882 \times 10^{-3}$ *and* $B = -0.103$, t_0*=0.4, 0.7, and 0.9.*

Similarly, if λ is fixed and ϑ varies from 0.35 to 0.45 in Table 3, the straight line is smaller than that of Figure 5 which has been shown in the Figure 9.

But for fixed time and different values of A & B which has been indicated by Table 3, the total energy is decreasing slowly from Figure 7 to Figure 9.

4. Conclusion

From the above tables, figures and discussion we conclude that the following results:

- When magnetic diffusivity is constant and kinematic viscosity is changeable then the Magnetic Prandtle number is proportional to the kinematic viscosity.
- In the absence of non-dimensional quantity the total energy of magnetic field fluctuation is decaying very rapidly.
- The magnetic field fluctuation of total energy is gradually increased at fixed time t_0= 0.4, 0.7, and 0.9.
- The energy decay is very small at constant time which has been shown in the Figure 6 to Figure 8.

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