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About the Closed Quasi Injective S-Acts Over Monoids

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Abstract: The aim of introducing and studying the notion of closed quasi injective S-act is to create a basis facilitate for the exchange ideas between module theory and act theory. As well as it represents a generalization of the quasi-injective act. The quasi-injective act was first introduced and studied by A. M. Lopez, Jr. and J. K. Luedeman, 1979. Then the author was one of the researchers which introduced several generalizations for this notion from several aspects because of its importance. More accurately, the contribution of this paper to the field of competence can be summarized into three points as follows: First: The possibilities for applying the topic of this article helps researchers about how can connect class of injectivity with its generalizations. Second: Study the topic of this article contributes to the improvement of the vision for finding the corresponding between acts theory and module theory. Third: This article has dealt with the important subject in the field of science and knowledge especially in algebra and can take it as a basis for future work for the researchers who work on algebra. Now, in this paper, the concept of closed quasi injective acts over monoids is introduced which represents a generalization of quasi injective. Several characterizations of this concept are given to show the behavior of the property of closed quasi injective. Relationship of the concept of closed quasi injective acts over monoids with Hopfian, co-Hopfian and directly finite property are considered. This work gives the answer to the question of what are the conditions to be met in the subacts in order to inherit the property of closed quasi injectivity. We obtained the main result in this direction in proposition (2.5) and proposition (2.6). A part of this paper was devoted to studying the relationship among the class of closed quasi injective acts with some generalizations of injectivity.

Keywords: Closed Quasi Injective Acts, Extending Acts, Continuous Acts, Noetherian Acts, Hopfian Acts

1. Introduction

Actions of a semigroup have always been interesting to mathematicians. From an algebraic perspective, a semigroup action is a generalization of the notion of group action in group theory. Besides, it is familiar that in the theoretical computer science and in algebra, an action of a semigroup on a set is a rule which associates to each element of the semigroup is a transformation of the set in such a way that the product of two elements of the semigroup is associated with composite of two corresponding transformations. The terminology conveys the idea that the elements of the semigroup are acting as transformations of the set. An important special case is a monoid action or act, in which the semigroup is a monoid and the identity element of the monoid acts as the identity transformation of a set. It is recognized that the theory of monoids and systems is a generalization of the theory of rings and modules, which has a number of direct applications in theoretic Computer science, Theory of differential equations and Functional analysis, etc.

Now, by a monoid S we always mean monoid with zero elements 0 and every right S-act M is unitary with zero element Θ which denoted by M_S. A right S-act M_S with zero is a non-empty set with a function f: $M \times S \to M$, $f(m, s) \mapsto ms$ such that the following properties hold: (1) $m \cdot 1 = m$ (2) $m(st) = (ms)t$, for all $m \in M$ and $s, t \in S$, 1 is the identity element of S. It is possible to find the S-act with several names as follows: S-systems, S-sets, S-operands, S-polygons, Transition systems, S-automata [1]. Note that we will use terminology and notations from [2-5] freely. For more details about injective acts and their generalizations we refer the reader to the references [6-8]. Familiar concepts are good and natural stations for starting. Recall that a non-zero subact N of M_S is intersection large if for all non-zero subact A of M_S, $A \cap N \neq \Theta$, and will be denoted by N is ∩-large in M_S. Equivalently, if for each $\Theta \neq m \in M_S$ there exists $s \in S$ such that $\Theta \neq ms \in N$ [9]. In this case, we call M_S is ∩-large extension of N. An S-homomorphism f which maps an S-act M_S into an S-act N_S is said to be split if there exists S-homomorphism g which maps N_S into M_S such that $fog=1_M$ [10].

Let A_S, M_S are two S-acts. A_S is called M-injective if given an S-monomorphism α: $N \rightarrow M_S$ where N is a subact of M_S and every S-homomorphism β: $N \rightarrow A_S$, can be extended to an S-homomorphism σ: $M_S \rightarrow A_S$ [11]. An S-act A_S is injective if and only if it is M-injective for all S-acts M_S. An S-act A_S is quasi injective if and only if it is A-injective. Quasi injective S-acts have been studied by Lopez and Luedeman [12]. A subact N of S-act M_S is called closed if it has no proper ∩-large extension in M_S that is the only solution of $N \hookrightarrow^{\cap l} L \underset{\neq}{\hookrightarrow} M_S$ is N=L [13]. An S-act M_S is said to satisfy C_1-condition if every closed subact of M_S is a retract subact of M_S. An S-act M_S is said to satisfy C_2-condition if every subact of M_S which is isomorphic to retract subact of M_S is itself a retract subact of M_S. An S-act M_S is called a CS-act or extending act if it is satisfied C_1-condition. An S-act M_S is called continuous if it is satisfied C_1 and C_2-conditions [14]. In early time we gave some generalizations of the quasi-injective S-acts, which represent generalizations of the doctoral dissertation of the author. Now, we adopt another generalization of quasi injective act which is C-quasi injective acts to characterize the behavior of the property considered under well-known constructions such as product, coproduct, and direct sum.

This article consists of three sections. The first one (section two) is devoted to introduce and investigate a new kind of generalization of quasi-injective S-acts, namely C-quasi injective over monoids. Certain classes of subacts which inherit the property of C-quasi injective acts were considered. Also, the characterizations of this new class of S-acts were investigated. An example was given to demonstrate C-quasi injective acts over monoids. Some known results on C-quasi injective for general modules were generalized to S-acts. In the second part (section three) has clarified the discussion for our results. The third part (section four) has clarified the conclusions of our work.

2. Main Results

Definition (2.1): Let M_S and N_S be two S-acts, N_S is called closed M-injective (for short C-M-injective) if for any homomorphism from a closed subact of M_S to N_S can be extended to homomorphism from M_S to N_S. An S-act N_S is called closed quasi injective if N_S is C-N-injective. A monoid S is called right closed self-injective if it is C-S-injective.

Remark and Example (2.2):

(1) Every quasi injective act is closed quasi injective (simply C-quasi injective), but the converse is not true in general, for example Z with usual multiplication is C-quasi injective Z-act, but it is not quasi injective.

(2) Obviously, definition (2.1) is up to isomorphism. This means that isomorphic act to C-quasi injective act is C-quasi injective.

Lemma (2.3): Let A_S and B_S are two S-acts. If f: $A_S \rightarrow B_S$ is isomorphism and X is ∩-large subact of A_S, then f (X) is ∩-large subact of B_S.

Proof: For each f (Y) which is any non-zero subact of B_S there exists a non-zero subact Y of A (Since f is isomorphism). As X is ∩-large subact of A, so we have $X \cap Y \neq \Theta$. Now, we must prove that $f(X) \cap f(Y) \neq \Theta$. Since f is monomorphism, so $X \cap Y \subseteq f(X) \cap f(Y)$, but $X \cap Y \neq \Theta$. Hence $f(X) \cap f(Y) \neq \Theta$ and f (X) is ∩-large subact of B.

Proposition (2.4): Let N is a closed subact of S-act M_S. If N is C-M-injective act, then any monomorphism from N into M_S split (in other words if N is C-M-injective act, then N is a retract subact of M_S).

Proof: Let α: $N \rightarrow M_S$ be monomorphism such that N is closed subact of M_S. Since N is C-M-injective, so there exists S-homomorphism β: $M_S \rightarrow N$ such that $\beta o \alpha = I_N$. This means that N is a retract subact of M_S, since $N \cong \alpha(N)$, so $\alpha(N)$ is a retract of M_S and f is split.

The following proposition give under which condition the subacts inherit the property of C-quasi injective.

Proposition (2.5): Let M_S is C-quasi injective act. Then every fully invariant closed subact of M_S is C-quasi injective.

Proof: Let N be fully invariant closed subact of M_S and let K be closed subact of N and let α: $K \rightarrow N$ be S-homomorphism. Since N is closed subact of M_S, it follows that K is closed subact of M_S by lemma (2.4) in A. shaymaa's study [14]. Then, by C-quasi injectivity of M_S, there exists β: $M_S \rightarrow M_S$ that extends α. Since $\beta(N) \subseteq N$ by hypothesis, so this means there exists σ from N into N which is the restrict of β and extends α. Thus N is C-quasi injective.

Proposition (2.6): Every retract subact of C-M-injective act is C-M-injective act.

Proof: Assume that an S-act N is C-M-injective and A is a retract subact of S-act N_S. Let X be closed subact of S-act M_S and f be S-homomorphism from X into A. Since N is C-M-injective act, so there exists S-homomorphism g from M into N_S such that $goi_X = j_A of$, where j_A is the injection map of A into N_S. Put $h = \pi_A og$, where π_A is the projection map of N_S onto A, then $hoi_X = \pi_A ogoi_X = \pi_A oj_A of = f$ and A is C-M-injective.

Proposition (2.7): Let M_S and N_S are two S-acts. If N_S is C-M-injective act, B is a closed subact of M_S, and then N_S is C-B-injective act.

Proof: Let X be closed subact of B, and f be S-homomorphism from X into N_S. Since N_S is C-M-injective, so there exists S-homomorphism g from M_s into N_S such that $goi_B oi_X = f$, where i_X, i_B be the inclusion map of X into B and B into M_S respectively. Put $h = goi_B$, then $hoi_X = goi_B oi_X = f$. Thus N_S is C-B-injective act.

Corollary (2.8): Let M_S and N_S are two S-acts. Then, N_S is C-M-injective act if and only if N_S is C-X-injective act for every closed subact X of M_S.

Proof: Suppose that N_S is C-M-injective act, by proposition (2.7), we have N_S is C-X-injective for every closed subact X of M_S. Conversely, since M is closed subact of M_S and by assumption, we have N_S is C-M-injective act.

Proposition (2.9): Let M_S be an S-act and $\{N_i \mid i\in I\}$ a family of S-acts. Then $\prod_{i\in I} N_i$ is C-M-injective act if and only if N_i is C-M-injective act for every $i \in I$.

Proof:⟹) Assume that $N_s = \prod_{i\in I} N_i$ is C-M-injective. Let X is closed subact of M_S and f is S-homomorphism from X to N_i. Since N_S is C-M-injective act then there exists S-homomorphism g: $M_S \to N_S$ such that $goi_X = j_i of$, where i_X is the inclusion map of X into M_S and j_i is the injection map of N_i into N_S. Put $h=\pi_i og$, where π_i is the projection map of N_S onto N_i. Then $hoi=\pi_i ogoi=\pi_i oj_i of =f$.

⟸) Assume that N_i is C-M-injective for each $i \in I$. Let X be closed subact of M_S and f be S-homomorphism from X into $N_S=\prod_{i\in I} N_i$. Since N_i is C-M-injective act, then there exists S-homomorphism β_i: $M_S \to N_i$, such that $\beta_i oi=\pi_i of$, so there exists S-homomorphism β: $M_S \to N_S$ such that $\beta=j_i o\beta_i$. We claim that $\beta oi=f$. Since $\beta oi=j_i o\beta_i oi=j_i o \pi_i of=f$, so we obtain $f=\beta oi$. Therefore N_S is C-M-injective.

Proposition (2.10): If M_S^n is C-quasi injective act for any finite integer n, then M_S is C-quasi injective.

Proof: Let M_S^n is C-quasi injective act. By corollary (2.8), M_S^n is C-M-quasi injective act. Since M_S is retract of M_S^n, so by proposition (2.6) M_S is C-M-injective. Thus, M_S is C-quasi injective act.

Recall that an S-acts M_i, $i\in I$ is called relatively C-injective acts if M_i is C-M_j-injective for all distinct i, $j \in I$, where I is the index set.

Lemma (2.11): Let M_1 and M_2 be two S-acts and $M_S=M_1\oplus M_2$. If M_s is C-quasi injective act, then M_1 and M_2 are both C-quasi injective act and they are relatively C-injective act.

Proof: Let M_s be C-quasi injective act, this means that M_S is C-M-injective. By proposition (2.6), we have M_1 and M_2 are C-M-injective acts. By corollary (2.8), we have M_1 (M_2) is C- M_2 (M_1)-injective act (since M_1 and M_2 are closed subacts).

Proposition (2.12): Every C- quasi injective act satisfies C_2-condition.

Proof: Assume that M_S is C- quasi injective act. Let f: B→A be an S-isomorphism, where A and B are sub acts of M_S and A is a retract of MS. Then A is C-M-injective act by proposition (2.6). Thus, by remarks and examples (2.2) (2), B is C-M-injective. Then, by C-M-injectivity of B the inclusion map i_B: $B \to M_s$ has left inverse g: $M_S \to B$ such that $goi_B=I_B$. Hence by proposition (2.4), i_B is splits and then B is a retract subact of M_S. Thus, M_S satisfies C_2-condition.

Now, we will study the relationship among C-M-injective act and injective act, extending act, continuous act.

Proposition (2.13): An S-act M_S is extending act (for short CS act) if and only if every S-act is C-M-injective.

Proof: ⟹) It is obvious.

⟸) Let N be a closed subact of S-act M_S. By hypothesis N is C-M-injective, so by proposition (2.4), N is a retract subact of M_S. It follows that M_S is CS-act.

Proposition (2.14): If every S- act is C-M-injective, then it is continuous.

Proof: Let M_S be S-act so by hypothesis M_S is C-M-injective act. By proposition (2.13), an S-act M_S satisfies C_1-condition and by proposition (2.12), M_S satisfies C_2-condition. Thus M_S is continuous act.

Recall that an S-act M_S is Noetherian if every subact of M_S is finitely generated. A monoid S is a right Noetherian if S_S is Noetherian. Equivalently, S is a right Noetherian if and only if S satisfies the ascending chain condition for right ideals (definition1.1.30) in book of M. Kilp, U. Knauer, and A. V. Mikhalev [4, p. 21].

Before the next theorem which is a generalization of theorem (1.1) in A. K. Tiwary, S. A. Paramhans, and B. M. Pandeya' study [15], we need the following theorem:

Theorem (2.15): [11] For a monoid S with zero, the following conditions are equivalent:

(1) Each direct sum of injective S-acts is injective.

(2) Each direct sum of weakly injective S-acts is weakly injective.

(3) Each injective S-act is countably Σ-injective.

(4) Each finitely injective S-act is weakly injective.

(5) S is Noetherian.

Theorem (2.16): The following conditions are equivalent for an S-act M_S, where S is Noetherian monoid:

(1) The direct sum of every two C-quasi injective S-acts are C-quasi injective acts.

(2) Every C-quasi injective act is injective.

Proof: (1⟹2) Assume that M_S is C-quasi injective act and E (M) is injective envelope of M_S. Then, by assumption $N_S=M_S\oplus E$ (M) is C-quasi injective. Consider the injection maps i: $M_S \to E$ (M), j_1: E (M) $\to M_S\oplus E$ (M), j_2: $M_S \to M_S\oplus E$ (M) and I_M: $M_S \to M_S$ is the identity map of M_S. Let π_M: $M_S\oplus E$ (M) $\to M_S$ be the projection map such that $\pi_M oj_2=I_M$. Now, $M_S\oplus E$ (M) is C-quasi injective, so this implies there exists S-homomorphism g: $MS\oplus E$ (M) $\to M_S\oplus E$ (M) such that $goj_1 oi=j_2 oI_M$, then $\pi_M ogoj_1 oi= \pi_M oj_2 oI_M$. Thus $I_M=\pi_M ogoj_1 oi$. Put $f=\pi_M ogoj_1$, then $I_M=foi$. Therefore M_S is a retract subact of E (M) and then it is injective.

(2⟹1) Let M_S and N_S be two C-quasi injective S-act. By (2) M_S and N_S are injective which implies that the direct sum of any two injective S-acts is injective whence S is Noetherian monoid by theorem (2.15) and then every injective is C-quasi injective. Therefore, the direct sum of two C-quasi injective is C-quasi injective.

It is clear that every co-Hopfian is directly finite, but the converse is not true in general (for this, assume that M_S is co-Hopfian and f, $g\in T=End$ (M_S) such that fog=I, then g is injective homomorphism. Since M_s is co-Hopfian, so g is isomorphism and thus there exists g^{-1}. Then, $f = fgg^{-1}=Ig^{-1}=g^{-1}$, so $gf=gg^{-1}=I$ which implies that M_S is directly finite). In the following proposition, we give a condition to be the converse is true:

Proposition (2.17): Every C- quasi injective act and directly finite is co-Hopfian.

Proof: Let f be an injective endomorphism of M_S and I_M is an identity homomorphism from M_S to M_S. Since M_S is C-M-injective act, so there exists a homomorphism g: $M_S \to M_S$ such that gof =I, since M_S is directly finite, so fog =I which implies that f is onto. Hence M_S is co-Hopfian.

The following proposition shows that the concepts of Hopfian, co-Hopfian and directly finite are coincide under C-quasi injectivity condition:

Proposition (2.18): Let M_S is C- quasi injective act. Then the following concepts are equivalent:

(1) M_S is Hopfian,

(2) M_S is co-Hopfian,

(3) M_S is directly finite.

Proof: (1→2) as every Hopfian is directly finite (For this if for any α, β∈End (M_S) and αoβ=I, then this means that α is surjective. Since M_S is Hopfian act and then α is isomorphism and β is inverse of α. Thus βoα=I which implies that M_S is directly finite act), so by proposition (2.17), M_S is co-Hopfian.

(2⟷3) By proposition (2.17).

(3→1) Let f be surjective endomorphism of M_S, then the inclusion map i: f (M)→M_S is isomorphism (since by proposition (2.17), M_S is co-Hopfian). Thus foi=$I_{f(M)}$. Again since M_S is directly finite, so iof =I_M (since f (M)≅ M_S). Thus f is injective and then it is isomorphism. Therefore, M_S is Hopfian.

3. Discussion

In this section, we clarify what's the meaning of the results which were obtained in this article. One of these results, it is proposition (2.4) where it was demonstrated that every monomorphism from closed subact into S-act is split when the subact is C-M-injective. As for proposition (2.5), it was clarified that closed subacts of C-quasi injective is C-quasi injective if they are fully invariant, while, proposition (2.6) explained that subacts of C-M-injective is C-M-injective if they are retracted. In addition, we were proved in proposition (2.9) that a finite product of C-M-injective acts is C-M-injective and the converse is true also, this means if the product is C-M-injective, then each M_i is C-M-injective. Besides, in proposition (2.10), we elucidated if the finite direct product is C-quasi injective, then each M_i will be C-quasi injective. Lemma (2.11) explained interesting result when S-act M_S has the form M_S= M_1⊕M_2 and it is C-quasi injective then each of the M_i (i=1, 2) will be C-quasi injective and relatively C-injective. Proposition (2.13) gave the coincide between extending act (CS-act), C-M-injective if every S-act is C-M-injective, while from proposition (2.14), we obtained that if every S-act is C-M-injective, then it will be a continuous act. Also, the theorem (2.16) showed essential identical between two important conditions which were a. direct sum of every two C-quasi injective S-acts is C-quasi injective acts b. every C-quasi injective act is injective, when a monoid S is Noetherian. Furthermore, proposition (2.17) clarified that every C-quasi injective act coincides with Co-Hopfian if one can add the directly finite condition to C-quasi injective act to be Co-Hopfian, while the proposition (2.18) was demonstrated that the concepts of Hopfian, Co-Hopfian and directly finite will coincide if the S-act is C-quasi injective.

4. Conclusions

From previous theorem, examples, remark, and propositions, we can pick out some senior points as follows: We obtained an interesting result in proposition (2.4) which was: a closed subact N of S-act M_S is a retract subact of M_S if N is C-M-injective. Also, proposition (2.5) gave the answer to the question raised early in the abstract which was: what are the conditions to be met in the subacts in order to inherit the property of C-quasi injectivity? In the proposition (2.6), we concluded the characterizations of C-M-injective act. From proposition (2.9) and proposition (2.10), we deduced that the direct sum of C-M-injective act is also C-M-injective and if M_S^n is C-quasi injective act for any finite integer n, then M_S is C-quasi injective respectively. As for the lemma (2.11), we got that if M_S= M_1⊕M_2 is C-quasi injective acts, then M_i (i=1, 2)) are relatively C-injective acts.

In addition, proposition (2.13) and proposition (2.14) gave the relationship among C-M-injective acts with CS-acts and continuous acts respectively where these propositions illustrate that if every S-act is C-M-injective, then it will be a CS and continuous act respectively. In the theorem (2.16), we elucidated important result which is the existence of Noetherian monoid was solved the problem of the identical between the following conditions, I. The direct sum of two C-quasi injective S-acts is C-quasi injective acts and II. Every C-quasi injective act is injective. Finally, the relationship among the notions, C-quasi injective, Hopfian, co-Hopfian, and directly finite was illustrated in proposition (2.17) and proposition (2.18).

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