
An Empirical Research on Mathematical Case Development in Mathematical Teaching in Liberal Arts College

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Abstract: In order to explore mathematics learning methods suitable for liberal arts students specially and the educational value of mathematics application cases developed by liberal arts students, an experiment of mathematics application case study was carried out in the first grade liberal arts undergraduate students. 205 mathematics application cases have been obtained, and case studies were used with the statistical summarization and analytical generalization methods. From 2009 to 2013, about 531 liberal arts freshmen in *statistical* course completed a total of 90 mathematics survey reports containing the university campus life (58 cases), professional related research (21 cases), social issues (11 cases) and so on. From 2014 to 2018, liberal arts students completed 115 mathematics application cases in *Applied Probability and Statistics* class which can be divided into 3 categories according to the degree of innovation: using mathematical knowledge to solve problems (79 cases), empirical survey (14 cases) and literature analysis (22 cases). The case topics selected by liberal arts students are all related to their own interests, and the innovative uses are for the vast majority. Conclusion: (i) writing a mathematics survey report is an appropriate way for students to apply mathematics; (ii) mathematics application is the fusion of interest and knowledge in probability and statistical area for liberal arts undergraduate students; (iii) the development process of mathematics application cases can demonstrate innovative thinking and application ability for liberal arts undergraduate students.

Keywords: Mathematics Teaching in Liberal Arts College, Mathematical Case Development, Liberal Arts Undergraduate Students, Learning Interests

1. Introduction

In China, most liberal arts colleges offer advanced mathematics as a compulsory course in general education. As for liberal arts students in advanced mathematics class, 70% of them only study the basic content of mathematics compulsory in high school. They do not study mathematics elective courses offered by high school for mathematics students. What is the attitude of mathematics learning for liberal arts students? HE et al. (2013) conducted a questionnaire survey of 970 freshmen in arts college, history and culture college, education college, political law college, social college, and journalism communication department in some university. The result is that 41% of liberal arts freshmen expressed dislike of mathematics. 54% of them are dissatisfied and have no interest in mathematics, and never expect to earn ability to learn mathematics; 34% of them do not agree with the value of mathematics for professional development and self-promotion

[1]. Therefore how to improve the liberal arts students' understanding of mathematics learning value is an issue to be solved.

Is there any use for liberal arts undergraduate students to study mathematics? Firstly let's look at the use of mathematics. In the advanced mathematics textbook [2], the mathematics application case was incorporated into the mathematics content as the knowledge background or example: population flow prediction, human resource allocation, fund operation principle, product cost and profit, lottery profit mode, insurance company's payment probability and risk decision making, etc. The application of mathematics is showed in different fields. And there are some works in other countries that explain the application of mathematics. For example, David Hand from the United Kingdom uses probabilistic principles in his work [3] to explain gambling, weather, air crashes, creative writing, the origin of life, and so on. Jeffrey S. Rosenthal from the United States has introduced the incidence

of lottery, opinion polls, insurance policies, thunder, murders, natural disasters and terrorism in plain, non-mathematical languages to show that probabilities is playing an important role in daily life [4]. Peter Olofsson from Sweden has given a brief introduction to some important principles and classic questions, and an interesting and rational introduction to probabilistic events that even sometimes violate intuition. This book avoids mathematical derivations and complex mathematical formulas. Instead, fun examples are used to show the role of probability in everyday life [5]. It can be seen that probability statistics can explain lots of phenomena in society and daily life, solve problems, and help liberal arts students understand that it is necessary to master mathematical tools.

Is it valuable for liberal arts students to study mathematics? David S. Moore and Williaml Notz from the United States points out in his statistics textbook for liberal arts students that statistics is part of the humanities and is not only derived from uncertain empirical data but also a pattern to think. His book with no complicated formulas and calculations focuses on the application of statistical concepts and their impact on daily life, public policy, and many other areas. It highlights the idea that statistical thinking is a universal analytical method applicable to various fields [6]. The idea of David S. Moore and Williaml Notz about the teaching of liberal arts statistics is that university liberal arts students should learn statistical thinking. But that's not enough, explore the use of statistical thinking to analyze and solve problems is importance too. The work of Darrell Huff in the United States reveals examples of misleading statistical information to falsify in media, allowing people to see the fallacies and pitfalls hidden in statistical methods [7]. Therefore, it is the ability to think independently and judge statistical errors correctly when hearing some media or the promotion of certain organizations. Yan (1999) elaborated the relationship between knowledge and innovation. It is believed that "knowledge innovation" is the innovation that plays a leading role in knowledge advancement and different innovations contain different knowledge. Knowledge can be participated in innovation by new models, methods and concepts. The innovation function of knowledge can only be realized by the power of practice. [8]. Fan & Li (2006) pointed out that the content of knowledge innovation includes knowledge creation and knowledge application. Knowledge innovation depends on the use of knowledge [9]. Students should become practitioners who use mathematical concepts and ways of thinking. On May 5th, 2010, the Ministry of Education of China reviewed and approved the core of the strategic theme in "National Medium- and Long-Term Education Reform and Development Plan (2010-2020)", which is to cultivate students' talents on improving the social responsibility, innovative spirit of exploration and practical ability to solve problems [10]. Therefore, the educational goal of liberal arts mathematics courses is to cultivate innovative thinking and practical ability. Zhang (2016) showed the reform path of university innovation education starting from the classroom teaching from the perspective of the internal unity of

knowledge application and innovation [11].

It is important to find out teaching approach to achieve the goal of cultivating innovative spirit and practical ability of liberal arts. Studies have shown that the learning engagement of college students has a positive impact on learning outcomes [12]. Kuh G. (2009) points out that the more students study a subject, the more they know about it, and the more students practice and get feedback from faculty and staff members on their learning and collaborative problem solving, the deeper they come to understand what they are learning and the more adept they become at managing complexity, tolerating ambiguity, and working with people from different backgrounds or with different views [13]. Moreover, college students' engagement in learning is defined in three ways: the first is behavioral engagement which includes involvement in academic and social or extracurricular activities and is considered crucial for achieving positive academic outcomes; the second is emotional engagement which encompasses positive and negative reactions and willingness to do the work; the final cognitive engagement which incorporates thoughtfulness and willingness to exert the effort necessary to comprehend complex ideas and master difficult skills. [14]. Liberal arts students put their actions, emotions and cognition into mathematics learning and may achieve good learning results. Therefore, it can be an exploration of mathematics curriculum to let students develop the mathematics case. Liberal arts students choose their own content to develop a case so that they may be more willing to invest in emotions; they are required to use mathematics knowledge learned in this semester so that they will also understand and apply knowledge more deeply so that the process of students' creating cases is also the process of developing students' innovative spirit and practical ability goals.

In summary, Chinese liberal arts college students have insufficient understanding of mathematics learning value. Through mathematics courses, liberal arts students can not only understand the usefulness of mathematics, but also learn to solve problems in mathematics thinking. In fact, one of the teaching goals of the general education courses is to cultivate the innovative spirit and practical ability of college students. However, there is little research on how to implement the overall goal of this education to classroom teaching. This study intends to guide liberal arts students to develop mathematics application cases independently so that students can not only understand the value of mathematics learning but also cultivate the ability to analyze and solve problems in mathematical thinking with innovative elements. Therefore, this research is very important and has practical significance. In this paper, liberal arts students were organized to use mathematics knowledge to develop mathematics application cases in the form of group cooperation. Since 2009, a total of 1,123 students have completed a total of 205 mathematics application cases, including article, video online, text report, etc. This paper intends to analyze the ways mathematics learning might be suitable for liberal arts students specially and the educational value of mathematics application cases developed by liberal arts students.

2. Research Design

2.1. Research Questions and Hypotheses

The research questions are:

(i) Can liberal arts mathematics curriculum from general education find an effective way for students to apply mathematical knowledge methods to solve problems? (ii) The needs and interests of liberal arts students are important in the process of mathematics learning, so how are they related? (iii) The ability to cultivate college students' sense of social responsibility and improve their innovative thinking and problem solving ability are the main goals of general education, therefore how can liberal arts mathematics curriculum achieve these training goals?

The research hypotheses are:

H1: Writing survey reports might be a quick way to apply mathematics for liberal arts students;

H2: Mathematical application might be the fusion point of interest and mathematics knowledge for liberal arts students;

H3: The process of developing mathematics application cases for liberal arts students can probably demonstrate their innovative thinking and problem solving ability.

2.2. Sample

The study is divided into two phases: the first phase from 2009 to 2013, and the second phase from 2014 to 2018.

The first stage (2009 - 2013), In order to let liberal arts students understand the application of mathematics in real life, the study lasts four years and has guided 531 freshmen in the Art College, History College, Chinese Language and Literature College, and Philosophy College. The *statistics* course is 36 class hours per semester, and the gender percentage of liberal arts students is that boys account for 19.8% and girls 80.2%.

In *statistics* course, students cooperated in groups, selected topics by interest and wrote about 90 research reports.

The second stage (2014 -2018), the students have taught are liberal arts freshmen in the Faculty of Education, and science students from the Institute of Education Technology is excepted. The total number is 592, boys accounted for 17.7% and girls 82.3%. In the first semester, students studied *Calculus* for 48 class hours and the mathematics application case study is implemented in the second semester in 48 class hours of *Applied Probability and Statistics*.

In order to further guide liberal arts students to expand the application scope of mathematics knowledge, students are encouraged to use concepts, principles and methods of mathematics to explain the phenomena and solve the problems in the real world. At the beginning, previous mathematics application cases were introduced and commented on the characteristics to the students, then put forward the requirements for the student development case. Student's work can be submitted at any time during the semester, so that when there are outstanding students' cases submitted, they would be showed in the whole class to play a guiding role for other students. For five years 592 undergraduate freshmen have

been instructed in Beijing Normal University (BNU), and they worked by groups to develop 115 mathematical application cases.

2.3. Research Methods

This study firstly determines the research questions and hypotheses based on the paradigm of exploratory case studies. Secondly, 205 mathematic application cases are analyzed by using the statistical summarization and analytical generalization methods [15], while the data are sorted, summarized, reduced, and described as distribution characteristics. The ways of data collection is that case development is arranged as a course assignment to students. Guidance would be provided to them based on the results of the case completion and propose the case development standards. The purpose is to ensure the feasibility of the development of mathematics cases for liberal arts students. After the students submitted the cases, they were summarized and classified by topic. Excellent works would be analyzed to improve previous case development standards. The amount of completed cases is 205, which developed by 1123 students from 2009 to 2018, 18.75% of boys and 81.25% of girls. Finally, the mathematics application cases are taken as analysis units so that the logic of cases and research hypotheses can be linked.

3. Study on Students' Mathematical Cases

3.1. Phase I (2009 - 2013)

Hypothesis 1: Writing survey reports might be a quick way to apply mathematics for liberal arts students.

In the first phase of the study, a total of 531 freshmen from the Art College, History College, Chinese Language and Literature College are required to work together to complete 90 survey reports in statistics class. Guiding steps were as follows. Firstly, introduce the published cases; Secondly, conduct a survey exercise and ask the students to develop cases. Finally, excellent cases are showed for these students. For example, from September 2009 to January 2010, a *statistical* class was taken at the Art College and introduced the case "The Influence of Communication on Children's Modernization" to let students understand the research background, theoretical discussion, research design, and the conclusion. Therefore students could think about research methods, understand the process and methods of investigating reports, and then do exercises for the designated topics such as "The future direction of graduate students from Art College in 2008". Then, the criteria for making the investigation report are proposed, and the students can complete the investigation report by self-selection in the form of group cooperation. Finally, students' excellent cases could be showed in front of the whole class, such as "the market demand and evaluation report of the movie *Avatar*". Since student's survey reports can be seen as a teaching exploration, the above work only takes 4

class hours, and the remaining 32 hours are mainly times for teaching statistical knowledge.

90 survey reports completed in the first stage covers three categories: "college life", "professional related research" and "social issues". The specific contents are as follows:

(i) University campus life (58 cases): diet, sleep, consumption, relationship, happiness, university library resource utilization, freshman student community choice, method and type of class notes, study time spending for a class, college students' career choices, employment rates, etc.;

(ii) Professional related research (21 cases): market demand of various films, music selection of contemporary people, surveys of literary works adapted into film, development direction of a college, "personal pressure" for all age groups, recognition of some digital cameras, attitude to the university's international exchange program, etc.;

(iii) Social issues (11 cases): students' recognition and acceptance of foreign holidays, students' awareness of intellectual property rights, the constellation view of college students, the public's understanding of popular culture, and the status of young people playing online games;

The above 90 survey reports show that liberal arts students can apply statistical knowledge to various fields, describe the phenomenon, reveal the reasons, and make judgments based on quantitative results. Here is an example about the research on market demand and evaluation of the film "Avatar" by the students.

In January 2010, when the film "Avatar" was released in China, a team consisted of 6 students majored in film and digital media went to the cinema to conduct a questionnaire survey of 63 people before and after watching the movie. The purpose of their survey is to combine the knowledge of film media majors to analyze the trends of modern film market and the audience's demand and evaluation of films, so as to make reference for their own professional learning and future development. The survey is aimed at audience's expectations about "Avatar", the influence of the film promotion, and the feeling after watching. Statistical results are as follows: Most of them chose high-level special effects production and an excellent sci-fi story. It is not difficult for people to come to the conclusion that the future of film development will be in the direction of high technology, and constantly improving and enriching the film story. In addition, it is found that the upgrade of film technology today such as IMAX's prospects after the promotion of "Avatar" is brilliant, which also means that people's viewing needs are increasingly strong. This case shows that using the statistical knowledge, analyzing the demand of the film market and speculating the development direction of film technology from major perspective can lay a professional research foundation for liberal arts students.

Whether the description of the overall 90 survey reports or the above description of "Avatar case" indicates that their case is successfully completed, which can indicate that the investigation report is an effective way for them: hypothesis 1 is established. Writing survey reports is a quick way to apply mathematics for liberal arts students, and to build bridge from statistical methods to students' interests. However, the

disadvantage is that although survey report is rich in content, there is limited knowledge of the application of statistics knowledge. Therefore, it is necessary to change single form of the survey report to explore how mathematics knowledge can be integrated with the interests of liberal arts students.

3.2. Second Stage (2014 - 2018)

Hypothesis 2: Mathematical application might be the fusion point of interest and mathematics knowledge for liberal arts students.

For the past five years, *Applied Probability and Statistics* course were taught in Faculty of Education. Students would be taught about how the examples in the book collect data, which formulas and theorems are used, what ideas can be expressed, and then ask them to make a case. The teams cooperated to submit mathematical application cases within a semester and record it in the mark at ordinary times. Liberal arts students participated in this study have developed a total of 115 mathematical application cases involving many probabilistic statistical knowledge, such as classical probability, geometry approach, conditional probability, total probability Formula, Bayes Formula, Bernoulli probability, binomial distribution, normal distribution, mathematical expectation, variance, central limit theorem; descriptive statistics, correlation analysis, sampling, point estimation, interval estimation, etc.

Most of the topics in 115 cases are related to the interests of liberal arts students. 115 cases were divided into three categories according to the degree of innovative applied mathematics. The first type is an innovative case of knowledge application, It contained 79 cases accounted for 68.7%. Students find problems, ask questions, and apply mathematics to solve problems, such as "Love mathematics more than intuition", "Which is more difficult to get access in Tsinghua University and 5 million dollars?", "Fan girl's drawing lots", etc. The second type is empirical investigation cases, It contained 14 cases accounted for 12.2%, such as "The power of big data -- revealing the truth of the statistical common value of 17.53%", "What affects the choice of our university study majors", "Single troubles", etc.; The third type is based on the analysis cases of the existing literature, It contained 22 cases accounted for 19.1%. such as "The reasonable use of mathematical expectation from the blood test of the United States World War II conscription examination group", "Analysis of the ancient Chinese military science" and so on. Among the above three types of topics, students applying mathematics knowledge are the vast majority, indicating that there is a lot of space for the application of mathematics for liberal arts students.

As can be seen from the above description, the survey questions selected by students are related to their own interests. It will be show this example case below to explain the case of "Fan girl's drawing lots".

Background: Yi is a famous Chinese singer and actor. In 2018, he is 18 years old, same age as freshmen fans in this study. A team of 6 fans made a mathematical application case below combined with the purchase of tickets online for the Yi

Millennium concert. The girl Xiao Meng is a fan of Yi and she wants to get up at 0 o'clock and participates in the lottery to get tickets online for the idol concert. The group mates helped her find the data and calculate the probability of the tickets being drawn. The number of tickets for the Yi concert tickets is: 1200 from the performance company, 100 from micro blogs of Yi, 300 from *Taobao.com*, and 200 from *Tmall.com*.

Set

$A = \{\text{Xiao Meng wins lottery}\}$

$B = \{\text{She draws tickets from the performance company}\}$

$C = \{\text{She draws tickets from Yi's micro blogs}\}$

$D = \{\text{She draws tickets from Taobao.com}\}$

$E = \{\text{She draws tickets from Tmall.com}\}$.

According to the data collected by the group students: (i) The number of participants in the company's official website is 30,158, (ii) The number of people participating in the Yi's micro blog is 910,000, (iii) The number of people participating in *Taobao.com* is 161,000, (iv) The number of people participating in the *Tmall.com* is 340,000.

Then

$$P(A|B) = 1200/30158 = 0.0398;$$

$$P(A|C) = 100/910000 = 0.0001$$

$$P(A|D) = 300/161000 = 0.0019$$

$$P(A|E) = 200/340000 = 0.0006$$

Then

$$P(B) = 0.3, P(C) = 0.1,$$

$$P(D) = 0.3, P(E) = 0.3.$$

Using total probability formula,

$$P(A) = \frac{P(B)P(A|B) + P(C)P(A|C) + P(D)P(A|D) + P(E)P(A|E)}{P(A)} \quad (1)$$

Based on (1), $P(A) = 0.0127$.

It is really "one hundred pick one". Xiao Meng sighed: every fan who chased the stars must be a winged angel, and chasing stars is not easy!

In the existing 205 mathematics application cases, all the subjects are selected by the liberal arts student group themselves. The mathematics teaching can arouse the students' awareness of mathematics application, they can find their own interest and mathematics knowledge fusion point, through young Students are interested in topics, reasoning, revealing the nature of things, and making predictions.

Hypothesis 3: The process of developing mathematics application cases for liberal arts students can probably demonstrate their innovative thinking and problem solving ability.

In the liberal arts mathematics teaching, some students can solve specific problems with their own unique methods, and many of them have questioned the general meaning of mathematical formulas in a criticized way. On the one hand, we affirm students' critical spirit; on the other hand, we

organize interactions among students, exchange the application of mathematical formulas in a wider field, cultivate their independent thinking, and improve their ability to solve problems.

For example, when student A reads liberal arts mathematics textbook, he finds that the example of solving with the Bayes formula can be solved by addition principle and multiplication principle. So A's research topic is "Is there a use of Bayes formula?" His problem-solving process shows that the answer is no. His critical thinking has resonated in the whole class. This resonance is beneficial to the development of students' critical thinking, but it is a crisis for mathematics teaching. Therefore, students further explore the relationship between the total probability formula, the Bayes formula, and the addition & multiplication principle. It's found that A's method can only solve individual problems, and Bayes formula can solve general and complex problems.

A group of students found that using the Bayes formula to interpret Aesop's fable "Wolf is coming" has a charm.

Set

$A_1 = \{\text{this child is considered to be credible}\}$,

$A_2 = \{\text{this child is considered untrustworthy}\}$,

$B = \{\text{This child is lying}\}$.

Suppose the probability that the villager trusts the child at the beginning is 0.8 (reader-defined), then the probability of not trusting the child is 0.2, i.e.,

$$P(A_1) = 0.8, P(A_2) = 0.2.$$

If the child is considered to be credible, the probability of lying is 0.1 (reader-defined), the probability of lying under the premise of being untrustworthy is 0.5 (reader-defined), ie

$$P(B|A_1) = 0.1, P(B|A_2) = 0.5$$

The villagers first heard the call for help and rushed up the hill to fight the wolf, only to find that the wolf did not come, and the child lied (Event B occurred). Available from the total probability formula

$$P(B) = P(A_1) \times P(B|A_1) + P(A_2)P(B|A_2) \quad (2)$$

The result is

$$P(B) = 0.8 \times 0.1 + 0.2 \times 0.5 = 0.18$$

Considering Bayes formula

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(B)} \quad (3)$$

The result is

$$P(A_1|B) = \frac{0.8 \times 0.1}{0.18} = 0.444$$

According to this known information, the villagers corrected the impression of the child's credibility. After the villagers had once, the probability of the child being trusted has been adjusted to new $P(A_1) = 0.444$, $P(A_2) = 0.556$.

Still use

$$P(B|A_1) = 0.1, P(B|A_2) = 0.5,$$

Based on (2), new $P(B)=0.3224$, and the villagers heard the cry for the second time and rushed to the mountain but found that the wolf still did not come, the child is still lying (Event B happened)., based on (3) new $P(A_1|B) \approx 0.138$.

The villagers have twice corrected the impression of the child's credibility, and the probability that the child is considered to be credible has been adjusted to new $P(A_1) = 0.138$, $P(A_2) = 0.862$.

Still use

$$P(B|A_1) = 0.1, P(B|A_2) = 0.5.$$

Based on (2), new $P(B)=0.4448$. When the child called "The wolf is coming" for the third time, based on (3), the credibility of the villagers to the child was new $P(A_1|B) \approx 0.031$.

This data shows that as long as three lies are made, the trust probability of villagers' in children is reduced from the initial 0.8 to 0.031, which is almost untrustworthy.

If Aesop's fables allow people to accept the values of honesty from the situation, then using the Bayes formula to interpret Aesop's fables can be seen numerically. After three times of being deceived, people's trust probability is reduced to almost zero. The liberal arts students' application of mathematics shows the development of critical thinking, innovative thinking and moral cultivation.

3.3. Discussion

The liberal arts students are rarely going to be mathematics teachers in the future, but will do research and analyze problems in mathematical approach. The previous separation of our university liberal arts mathematics teaching and mathematics application leads to the questioning of mathematics valued when liberal arts students studied mathematics, and then in the subsequent study work they also "regret when the knowledge should be used." We should change our mind and guide the liberal arts students to develop their own mathematical application cases, analyze and solve problems, and form a mathematical way of thinking. The educational value of mathematics learning is self-evident. Plus, college mathematics teachers should also give liberal arts students technical support when developing mathematical cases.

All of the liberal arts students involved in BNU need to take the final exam so that they must master the basic mathematical formulas and principles. One doubt is that are learning mathematics knowledge and developing mathematics application cases are supplement each other for liberal arts students or increase the burden on students? This issue deserves further investigation and interviews.

4. Conclusions

4.1. The Conclusions of This Study

This study has reached the following conclusions during 9

years:

(i) Writing survey reports is a quick and appropriate way to apply mathematics for liberal arts students, and is also a bridge connecting mathematics knowledge with the interests of liberal arts students. The investigation report of liberal arts students can radiate to all aspects of university and social life. Through survey and statistics, they can use objective and rational attitudes, precise and quantitative mathematical methods, criticize open minds, and reveal the essence and universal laws of things.

(ii) Mathematical application is the fusion point of interest and mathematics knowledge for liberal arts students. Liberal arts students confirm their own viewpoints through survey statistics, data editing, mathematical reasoning and calculations, solve real problems, explore the essence of things, explore the true meaning of human nature, and predict a bright future. The age and interest fields of university liberal arts students are unique. Promoting the integration of students' interest and mathematics is the breakthrough point of mathematics teaching in university liberal arts.

(iii) The process of developing mathematics application cases for liberal arts students demonstrates their innovative thinking and problem solving ability. In the past 9 years, 205 applied cases in this study have presented from the topic of college students, to the use of literature, film and television, and online materials by liberal arts students, as well as texts, performances, reality TV videos, etc. The improvement of innovative thinking, problem-solving skills and the maturity of scientific research methods are well performed.

4.2. Limitations and Future Directions

Liberal arts students participated in this study have limited study majors, and the research objects are limited to Beijing Normal University. Whether there are limitations in the promotion of research results remains to be further investigated.

Research direction: In the past 9 years, liberal arts students have selected some development cases, but the contents of cases are scattered. This study intends to take following steps. First, teachers and students will discuss several general directions of mathematics application cases in cooperation. Then several groups will develop and apply mathematics problem solving cases from different sides in a certain direction, and it's expected the mathematics application case developed by university liberal arts students can express their ideas so that their works can not only improve themselves, but also spread to outside of the curriculum.

In the future, it will be obtained interview datum from students to understand their ideas in details. Further, it will be assessed whether this method can be applied in other universities and even in other countries. It is also appropriate to develop an appropriate evaluation standard of mathematics application cases for liberal arts students to play the guiding, stimulating, diagnostic and adjustment functions of evaluation.

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References

- [1] He Sui, Hu Dian-shun, LI Shu-gang. (2013). "Present Situation and Countermeasures of Mathematics Teaching in University of Liberal Arts". *Journal of Mathematics Education*. 1, 12.
- [2] Liu Jing-li (2016), "Advanced Mathematics for Liberal Arts (Edition 2)". Beijing: *Beijing Normal University Press*.
- [3] Hand, D. J. (2014). "The improbability principle: why coincidences, miracles, and rare events happen every day". *Scientific American/Farrar, Straus and Giroux*.
- [4] Rosenthal, J. S. (2006). "Struck By Lightning: The Curious World of Probabilities." *National Academies Press*.
- [5] Olofsson, P. (2015). "Probabilities: the little numbers that rule our lives". *John Wiley & Sons*.
- [6] David S. Moore, Williaml. Notz. (2014). "Statistics: Concepts and Controversies 8th Edition". *Worth Publishers*.
- [7] Huff D. (1993). "How to lie with statistics". *WW Norton & Company*.
- [8] Yan Xiao-feng. (1999). "Knowledge and Innovation". *Seeking Truth*, 6, 38-42.
- [9] Fan Zhi-ping, LI Shen-jie. (2006). "Connotation of Knowledge Creation and Knowledge Innovation and Relation Between Them". *Journal of Northeastern University (Social Science)*. 2, 102-105.
- [10] Ministry of Education of the People's Republic of China. (2010). "National Medium- and Long-Term Education Reform and Development (2010-2020)".
- [11] Zhang-qiong. (2016). "Knowledge Applying and Cultivation of Innovation Ability—On Professional Curriculum Reform in University". *Journal of Higher Education*. 3, 62-67.
- [12] Ernest Pascarella. (2010). "How Effective Are the NSSE Benchmarks in Predicting Important Educational Outcomes?". *Change*, January/February, 16-22.
- [13] Kub G. (2009). "The National Survey of Student Engagement: Conceptual and Empirical Foundations". *New Directions for Institutional Research*, 5.
- [14] Jennifer A. Fredricks, Phyllis C. Blumenfeld and Alison H. Paris (2004). "School Engagement: Potential of the Concept, State of the Evidence". *Review of Educational Research*, 74 (1): 60.
- [15] Yin, R. K. (2009). "Case study research: Design and methods (applied social research methods)". *London and Singapore: Sage*.